Title
Learning 10s Horizontally Improves First Graders’ Estimation of Numerical Magnitudes

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Publication Date
2016

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Santa Barbara

Learning 10s Horizontally Improves First Graders' Estimation of Numerical Magnitudes

A dissertation submitted in partial satisfaction

of the requirements for the degree

Doctor of Philosophy

in

Education

by

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September 2016

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Acknowledgement

I would like to express my deep gratitude to my advisor, Prof. Okamoto Yukari, for her excellent guidance, caring and patience, and providing me with an excellent atmosphere for doing research. I would also express gratitude to Professor Michael, Gerber. In the course ED176B Practicum in Individual Difference, the undergraduate students worked with me conducting the instructional interventions. The dissertation project would not had been gone so successfully without his enormous support. I would like to thank Professor Mary E Brenner, for guiding my research and patiently correcting my writing. Her wisdom, knowledge and commitment to high standards for the dissertation inspired and motivated me.

I would like to thank Shane Weafer, the ED 176B teaching assistant, who offered me great assistance in the process of collecting data, and working on training the tutors. I also want to thank my cohort member Patrick Pieng and Yeana Lam who discussed and addressed the methodology questions on the paper.

I would also like to thank my parents. They are always supporting me and encouraging me with the best wishes. Finally, I would like to thank my husband, Xiaofeng Liu. He is always there cheering me up and standing by me through the good times.

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Abstract

Learning 10s Horizontally Improves First Graders' Estimation of Numerical Magnitudes

by

Yu Zhang

This paper examined the number magnitude understanding and estimation of elementary students. Children as early as kindergartner differ in their understanding of numerical magnitudes. Research has found that teaching linearity of numbers was effective in improving children's estimation of numerical magnitudes (Booth & Siegler, 2008). Extending this line of work, I examined two different ways that linearity of numbers is taught: multiple 10 blocks and bundles. There were 58 kindergarten and first-grader students from Southern California randomly assigned to one of the three instructional contexts: blocks, bundles and control condition. I found that the students using the combination of base-10 and unit blocks gained more from the instruction than the other two groups. The students in the blocks group presented linearity for mental number line after the instruction. However, those in bundle and control groups could not.

Key words: numerical representation, cognitive development, core intervention model

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Chapter 1

Introduction

1.1 Statement of the Problem

Young children begin acquiring single digit numbers counting one by one, "One, two, three, four, five...” However, when it comes to multi-digit numbers, counting one by one becomes much fuzzier, which causes error and even frustrates children (Dehaene, Dehaene-Lambertz, & Cohen, 1998, p. 373). Children cannot connect the number value to quantity concepts in a meaningful way (Case et al., 1996). It takes a long time for them to link the multi-digit number name to the corresponding quantitative meaning. For instance, children might be able to recite large numbers of ten, hundred, thousand, but have trouble representing the quantities these numbers represent. This phenomenon was observed in several other measures of numerical magnitude understanding (Booth & Siegler, 2006). Especially given the interference of the spoken English number words that do not follow the base-10 system (e.g., 11 is eleven, not ten-one), English-speaking young children have difficulty building multi-digit numerical representations in comparison to East Asian children who learn that, for instance, 11 is ten-one (Miura, Kim, Chang &Okamoto, 1998).

Use of number lines may be an effective way to help children link multi-digit number words to their corresponding magnitudes. For instance, children may be asked to place 43 and 56 on the 0 – 100 number line. If they mark 43 on the left of 56, one could infer that they understand 56 is larger than 43 (Dehaene et al., 1998). From a developmental perspective, Case and Okamoto (1996) postulated that children by 6 years old develop a strong sense of single-digit number yet lack the ability to represent two-digit numbers. By 8−year-olds, however, children were expected to represent two-digit numbers. They further postulated that the majority of 10-year-old children should be able to generate linear representations of numbers beyond two-digit numbers. Empirical evidence supported their theory of children's development of single- to multi-digit number understanding.

Not all children fit the pattern of development described above. To help those children who are behind their peers, Ramani and Siegler (2008) developed a linear board game. In this game, children move a token from the start to finish with ten squares between them (1 to 10). Siegler demonstrated that kindergarten children who played this board game improved their understanding of single-digit numerals. However, things get complicated when it comes to two-digit numbers. Just counting one at a time is not effective in developing two-digit number magnitudes.

Understanding number magnitudes is essential to learning mathematics during elementary school years in the U.S. (NCTM, 2000). The National Mathematics Advisory Panel (2008) indicated "[p]oor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable" (p.27). It is important that the curriculum foster strong number sense in all children. One way to accomplish this is to provide children opportunities to link numerals and their number magnitudes. Use of number lines has been found effective for this purpose (Siegler et al., 2012). Teachers often use various concrete manipulatives to teach children number magnitudes such as Cuisenaire rods and bundles of sticks. For instance, in a common mathematics classroom, second graders are asked to make 34 by making three bundles and four sticks or using three base-10 blocks and four unit blocks.

Research has shown that young children's ability to represent numerical magnitude was associated with their middle and high school achievements in the longer term (The

National Mathematics Advisory Panel, 2008). Furthermore, children's understanding of number magnitudes and their learning activities were closely related to both arithmetic proficiency and their mathematics SAT score (Booth & Siegler, 2008). In addition, Opfer and Siegler (2007) found that using linear representation to estimate two-digit numbers was crucial to the estimation ability and future mathematics skills of grade 4 for two-digit number magnitudes.

1.2 Instructional Methods to Improve the Acquisition of Number Magnitudes

In the past decades, researchers and teachers have used many instructional methods and tools to promote in the number estimation learning process. As mentioned earlier, Siegler and his colleagues carried out an instructional intervention to assist preschool and kindergarten children who were behind their peers in fostering number magnitudes (Siegler & Thompson, 2014; Siegler, Thompson, & Opfer, 2009). Siegler and colleagues (2013) extended the 0-10 board game idea to designing a 10 x 10 board game. The numbers from 1 to 100 are written in ten rows, with the bottom row showing 1 to 10 from left to right and the top row showing 11 to 100. Laski and Yu (2014) used this board game with young children who showed improved understanding of two-digit number magnitudes. Thus, this could be a promising way to teach two-digit numbers. The accurate estimation of numerical magnitudes plays a pivotal role in the development of numerical knowledge. It seems plausible that understanding base-10 knowledge is important in acquiring number magnitudes. With improved enumeration and counting skills, students get a better sense of 10s and 1s embedded in specific two-digit numbers (Fuson, 1990). Therefore, early acquisition of place value concept should help young children better understand two-digit number magnitudes (Miura & Okamoto, 1989).

Repeatedly studies have shown that using manipulatives for teaching regrouping and base-10 concepts is effective. Manipulatives gained popularity in current elementary school mathematics with the introduction of a variety of concrete materials, including base-10 blocks, Cuisenaire rods, chips for trading, base-10 blocks, and Unifix cubes, to name a few. They have been proven to improve the base-10 understanding and grouping by 10 abilities (Kilpatrick, J., Swafford, J., & Findel, B., 2001). When students use manipulatives, they need to be helped to see their relevance for linking concrete representations to appropriate symbolism and mathematical concepts and operations. They require careful use over sufficient time to allow students to build meaning and make connections.

Another method that has been used to teach two-digit numbers or place value is the bundling of items to show groups of tens (Hiebert & Wearne, 1992). Base-ten concepts (ones, tens, hundreds) are frequently modeled with bundles and blocks. Bundles represent a common and effective instructional approach among the elementary mathematics classrooms. Use of such manipulatives is widespread and often acclaimed as beneficial for prospective teachers (Van de Walle, 1994). The first and second grade mathematics teachers used the bundles illustrating vividly base-10 concepts under the instructional circumstance in which such particular models and manipulatives can be used for grouping-by-10 purposes. A collection of templates, activities, and games that accompanied with the bundles and sticks are widely used by the mathematics teachers. The teachers selected multiple methods to solve problems and provided with scaffolds the students to connect multiple representations in meaningful ways.

NCTM's Principles and Standards for School Mathematics (2001) strongly support the effectiveness of these manipulatives, including bundles. Common Core State Standards suggests that first graders should know that “10 can be thought of as a bundle of ten ones—called a ‘ten’” (p.15). Hiebert and Wearne (1992), for example, found that first graders using bundles and sticks to count and represent two-digit numbers performed higher arithmetic results than those who did not. They concluded that using bundles and sticks could greatly promote the first graders’ two-digit number representation and understanding.

Hiebert and Wearne (1992) indicated that with the bundle manipulatives, the students played the games to bundle 10 sticks or coffee stirrers with a rubber band to make “ten” for a while. As students learnt how to carry out an operation such as two-digit addition and subtraction (for example, 43 – 19), they typically progressed to the more mathematical procedural. For example, an initial procedure for 43 – 19 might be to use bundles of sticks. Begin with 4 bundles of 10 sticks along with 3 individual sticks. Because the students could not take away 9 individual sticks directly from 43, they opened one bundle to made 13 individual sticks and then left 3 bundles. They took away one bundle from the 3 bundles (corresponding to 30 subtracting 10) and then 9 sticks (corresponding to 13 subtracting 9). The number of remaining sticks—2 bundles and 4 individual sticks, or 24—should be the answer. This procedure involves applying the grouping-by-10 procedures which, over sufficient time, allowed students to build meaning and make connections. Therefore, they concluded that bundles are the salient example of how materials are used for regrouping concepts. Children learned to interact with this specific representation to fully understand the underlying mathematical concept.

1.3 Purpose of the Study

The primary objective of this study was to improve kindergarten and first-grade students’ understanding of two-digit number magnitudes. To do so, I used two instructional approaches to teaching two-digit number magnitudes. One method was derived from the result of my pilot study that used base-10 and unit blocks to show two-digit numbers horizontally. Another method was derived from research on bundles (e.g., Hiebert & Wearne, 1992) as described above. Bundles were chosen for the present project because they are an effective tool for teaching regrouping although they do not have a clear linear structure.

In contrast, base 10 blocks, Cuisenaire rods, and Unifix cubes could be used by teachers and interpreted by students as supporting a linear structure. These bundles are an apt choice for the present study, which provides a clear contrast to my other intervention. Briefly, my pilot study connected a method of representing base-10 number system from the work of Miura and Okamoto (1989) and the work of Siegler and his colleagues on linear representations of numbers (e.g., Ramani & Siegler, 2008). More specifically, I created three instructional conditions for first-graders to encode two-digit numbers. All three conditions used commercially available base-10 blocks. One group of children learned to use both 10 and 1 blocks to construct two-digit numbers (multiple 10 blocks condition). Another group of children used one 10 block and multiple 1 blocks (single 10 blocks condition). And yet another group used only 1 blocks (multiple 1 blocks condition). I predicted that children learning to construct numbers linearly using a precise combination of 10 blocks and 1 blocks would learn to encode two-digit numbers more accurately. The results indicated that first graders in the multiple 10 blocks condition generated more accurate estimation on the 0−100 number line compared to the other two groups. For the present study, therefore, I selected and improved upon the method used for the multiple 10 blocks condition.

I postulated that the effect of base-10 understanding could be comparable between the block and bundle groups. However, the block group could achieve higher estimation results than the bundle group, which would be attributed to the more explicit linear clues. This present study was expected to improve the existing mathematics curriculum of elementary mathematics in public schools. The educational practice of playing number line games and building blocks would be an ideal pedagogical notion. This simple and inexpensive way could encourage student efforts and narrow the performance gap among kindergarteners and first-grade students, between those from low-income families and middle-to-high income families.

Chapter 2

Literature Review

This chapter discusses the rationale for this study. There is a theoretical drive to know about children's mental representation of number magnitudes among kindergarten and first-grade students. Young children often have been found to have learning difficulties for large number magnitudes, such as two-digit number magnitudes. Thus it is imperative to know how to effectively teach them to understand and represent two-digit numbers accurately. In the body of literature, there are two major lines of work that account for children's difficulties with two-digit number magnitudes learning. First, the children estimate the number magnitudes starting with the logarithmic pattern. When plotted on the 0–100 number line, the placements of the two-digit numbers present the logarithmic trend. The logarithmic number line impacts the accuracy of estimation. However, the older children, who estimate number accurately, present the linear representation of two-digit numbers on the 0–100 number line. The number representation presents the linear pattern when plotting the numbers on the 0–100 number line. The number representations of young children would go over from logarithmic to linear patterns during the developmental trajectory. Therefore, implementing linear principle in the number estimation learning is expected to enhance the children's cognitive development of number knowledge. In the literature review, I explain why and how to design and use the new manipulatives under the linear principle to support two-digit number magnitude learning in elementary mathematics classrooms. On the other hand, it has been long believed that acquisition of the base-10 system – the system of tens and ones – would improve children's estimation of two-digit number magnitudes. This suggests that students' learning of numbers with a combination of tens and ones would enable their accurate representation of numbers. Therefore, the present literature review combines the two lines of work to improve the implementation of a number line game. The theoretical framework demonstrates the research potential of representing 10s horizontally as a way was expected to promote kindergarten and first-grade students' numerical estimation. In addition, it is assumed that instructional intervention using such manipulatives of collections of base-10 units and unit in linear principle would enhance the instructional effect of number knowledge.

Numerical estimation is important for kindergartners and first graders. Starting in kindergarten, the Common Core State Standards (2013) required students to use visual models, objects, fingers, and mental images to represent number and to acquire number concepts at K-2 Grade. By the third-grade, students were expected to use visual models to solve two-digit number problems involving addition and subtraction. A previous study conducted by Korb (2007) considered the relationship between solid number knowledge and ability with multi-digit number estimation. The findings suggested that students who lacked a deep understanding of number magnitudes might encounter difficulties when solving numerical problems.

The standards and principles of the National Council of Teachers of Mathematics (NCTM) (2000) emphasized the importance of understanding numerical magnitude for preschool to grade 2 students. "Number magnitude is an essential feature for algorithms. Use of the algorithms not only depends on the automatic recall of number facts but also reinforces it. Far too many middle and high school students lacked the ability to accurately compare the magnitudes of such numbers" (NCTM, 2000, p.62). NCTM also acknowledged that estimation skills of the young children provided a solid foundation for future mathematics success not only for individual differences in computational skills (Dowker, 2005; Laski, Jor'dan, Daoust, & Murray, 2015; Laski & Siegler, 2014; Gilmore, McCarthy, & Spelke, 2007), but it was also closely related to individual differences in arithmetic, estimation proficiency and precision, magnitude comparison, numerical categorization, and overall mathematics achievement test scores (Booth & Siegler, 2008; Ramani & Siegler, 2011, 2014). Strong number sense also includes the ability to estimate the order of magnitudes to support computational skills. Therefore, mathematics instruction has to foster the development of numerical representation that draws from a robust understanding of numerical magnitudes. Furthermore, recent studies have shown that a child's understanding of numerical magnitudes is a strong predictor of later mathematics performance (Siegler & Booth, 2008).

The National Mathematics Advisory Panel (NMAP) (2008) has also addressed the importance of understanding numerical magnitudes. It reported that understanding number magnitude is essential to acquiring algorithms of addition and subtraction in middle and high schools. Students lacking this ability were unable to compare magnitudes of two or three-digit numbers, which interfered with their learning algorithms, as well as using strategies to verify solutions (Siegler & Booth, 2004, 2008).

2.1 Children's Estimation of Whole Number Magnitudes

Dehaene, Bossini, and Giraux (1993) examined 21 French children to find out how they conceptualized number magnitudes. They assumed that most children's mental representation of numbers on the number line followed the left-to-right track. The participants were assigned with some simple arithmetic operation, 2 + 3 =? They were asked to represent the numbers on the 0–10 number line. They observed that children compared the number magnitudes in terms of the number positions on the 0–10 number line. For instance, they determined that the number eight was larger than the number "seven," but smaller than the number "nine." They made correct judgments by retrieving that "eight" was on the right side of "seven." and on the left side of "nine." Dehaene, Bossini, and Giraux (1993) concluded that the children's ability for mentally representing single-digit numbers was compatible with their arithmetic ability within the range of 10. He claimed the importance of the mental number line on children's initial numeracy.

Young children (e.g., 3−5 years old) usually have difficulty representing number magnitudes. The disparities of estimation performance among children have been found as early as preschool and kindergarten (Dehaene, Lambertz, & Cohen, 1998). Young children usually found it difficult to understand number magnitudes by their numerals when they recited and learned large numbers. Dehaene (1998) suggested that preschoolers depend on a hypothesized number line to represent numbers larger than 5. They retrieved symbolic numbers from the number line to understand their quantitative meanings. For instance, they identified the quantitative meaning of the number "eight" by understanding that it was in between the number "seven" and "nine" on the number line. Students' development of mental number lines was also found to correlate to their mathematical development.

In studies of first graders' numerical magnitude, the students' representations were examined by mental number-line tasks (Opfer & Siegler, 2007). The first graders were asked to plot the two-digit numbers on the 0−100 number line. The estimated numbers were measured against the actual number magnitudes to obtain the estimation accuracy results of the young children. Opfer and Siegler (2007) intended to know the relationship between the number estimation ability in early years and arrhythmic performance. The numerical understandings of first graders were found closely correlated to arithmetic ability in both the pretest and posttest. After controlling for the factor of average mathematics scores, students with a higher estimation accuracy results were found to have performed higher in the addition and subtraction problems as well as on arithmetic problem solving. Therefore, Opfer and Siegler concluded that the children with a greater knowledge of numerical magnitudes of numbers had the potential to acquire the arithmetic skills.

Knowing number magnitudes is essential to many mathematics skills. However, the disparities in representing two-digit numbers have been reported at the entry-level of elementary school (Booth & Siegler, 2006, 2008; Dehaene, Dupoux, & Mehler, 1990; Siegler & Mu, 2008). When reflecting the numbers on the linear number line, English-speaking children followed the left-to-right rules (Dehaene, 2011). Young children (5–7 years old) usually found it hard to understand large number magnitudes, such as two-digit numbers. A majority of young children could not associate numerical quantities with the semantic meanings of numbers when reciting and learning large numbers (Wynn, 1992). The studies of Siegler and Thompson (2014) indicated that when reciting the numbers up to 100, the children did not understand its quantity as precisely as the numbers up to 10. Reflected on the number-line estimation task, they might put 100 on the right side of 10 to indicate that 10 was smaller than 100. However, they could not estimate the number magnitudes as accurately as the older children (8–9 years old). Younger children estimated 100 close to the positions of 11 or 13 on the 0–100 number line.

2.1.1 Logarithmic-to-linear cognitive shift. Siegler and Booth (2004) conducted a series of number-line estimation tasks including kindergarten, first, second, fourth, and six grade students. In each grade level, there were about 20 students who participated in the study. Siegler and Booth (2004) utilized the number-to-position (NP) method to test which model best fit a child's mental number line. They used a horizontal line, which was unmarked except for the start and endpoints (e.g., 0−100, 0−1,000, or 0−10,000) (Siegler, Thompson, & Opfer, 2009). The children were asked to put the given numbers on the 0–10, or 0–100 or 0–1,000 number line to indicate where the number should be. The position of the numbers indicated the number magnitudes of the given numbers. The target of NP task was a mark present on the line for which a number was supplied requiring the student to make a corresponding mark on the line. For instance, for the 0−100 number-line estimation tasks, when asking the participants to estimate the position of the number "26," the experimenter would ask while pointing to the physical 0−100 number line: "Here is '0' and here is '100', where do you think the number 26 should go?" (Ashcraft & Moore, 2012; Booth & Siegler, 2006, 2008).

A linear regression equation was obtained to identify the best-fitting model of the students' mental number line on the 0−100 number line ($y = ax + b$) (Siegler & Booth, 2004). The best-fitting model of mental number line patterns was decided by the two indexes of R^2 and slope. For instance, when $R^2_{linear} > R^2_{log}$, the best-fitting model of an individual's mental number line was identified as a linear pattern. On the other hand, if $R^2_{linear} < R^2_{log}$, the best-fitting model of his/her mental number line was a logarithmic pattern. If $R^2_{linear} = R^2_{log}$, his/her mental number line could fit either model. Slope "a" was another index of linearity on the number line ($y = ax + b$). It showed the slope of the regression curve. Slope "a" was a composite index of the linearity of the 0−100 number line with R^2. A large R^2 (i.e., R^2 close to 1.0) together with a slope closer to 1.0 was a strong case for a linear-fitting model of an individual's mental number line. However, a small slope (closer to 0) might impact the linearity of an individual's mental number line even though R^2 was large. The ideal linear model of the 0−100 number line could be $y = ax + b$ with $R^2_{linear} = 1.00$ and $a = 1.00$.

This developmental change from logarithmic to linear understanding was often referred to as the logarithmic-to-linear shift. Siegler and Booth (2004) found that the more linear the students' assessed number line was, the more accurate their estimation ability was considered to be. They (2004) found that at each developmental phase, a student's mental representation started from a logarithmic representation. Resembling the patterns of the preschoolers' estimates, which follow a logarithmic-to-linear function on a 0−10 number line (White & Szucs, 2012), the kindergarteners usually started with a logarithmic representation on the 0−100 number line. However, they transitioned to a linear representation in the second grade. The general developmental pattern was expected to repeat for each multi-digit number range as found on the 0–100 number line. The fourth graders were expected to show a logarithmic representation on the 0−1,000 number line; and the six graders on the 0−10,000 number line. However, the findings indicated that the estimation of fourth and fifth graders fit the linear pattern on the 0−1,000 number line. Additionally, the best fitting model for sixth graders was linear on the 0−10,000 number line.

Booth and Siegler (2006) examined this issue with first-graders who were in the crucial developmental stage of logarithmic-to-linear transition from logarithmic to linear representations of the 0–100 range. The students were assessed to discover their developmental levels of estimation and arithmetic ability. They were presented with a similar number-line estimation task at pretest and posttest. They were also examined with addition problems (e.g., 1 + 4 and 5 + 4 to 38 + 39 and 49 + 43). The participants were presented with each of these problems three times, with feedback regarding the correct answer being provided after each presentation. All of the sums were between 0 and 100. On each item, the experimenter read the problem aloud and asked the child which of the three choices was closest to the answer (e.g., “Is 34 + 29 closest to 40, 50, or 60?”) They children were expected to acquire the two-digit number and identify the number magnitudes of each item. The levels of estimation accuracy were very comparable between the first graders (percentage of absolute error = 14%) and second graders (PAE = 10%). Both of them were higher than the kindergartners (PAE = 24%). There were two reasons to account for this Booth & Siegler, 2006). The kindergartners and the first graders were a better fit of the logarithmic function on the 0–100 number line (R^2 = 0.89) rather than the linear function (R^2 =0.62). However, students in the second grade produced estimates that fit a linear distribution (R^2 =.96). The kindergartener also presented the estimation errors that were significantly higher than the second graders, which was closely related to the linear representation levels of the mental number line. In addition, the students’ arithmetic abilities were also related to the estimation level and linear representations. The more accurate their estimated numbers, the higher scores they received on the arithmetic tasks.

Although the logarithmic-to-linear representation has been widely identified and studied, some researchers have switched attention to an alternative explanation, emphasizing a multi-linear hypothesis. Ebersbach, Luwel, Onghena, and Verschaffel (2008) provided evidence for a two-segment model with the break point at 10. There were 60 participants, ranging from 5–9 years old involved in the study. All the kindergarten and first grader students were presented with the number-identification questions and number-line estimation tasks. The number-line tasks were broken into two parts with 0–10 and 10–100 number-line estimation. The findings indicated that the linearity R^2 was remarkably different on the two number lines. On the 0–10 number line, the slope seemed steep, which indicated the exaggerated number magnitudes of single-digit numbers (e.g., the estimated number magnitudes were larger than the actual numbers). In contrast, on the 10–100 number line, the number line slope was flat with R^2 and slope close to 0, which indicated diminished number magnitudes on the range 10−100: the estimated number magnitudes were smaller than the actual numbers. Ebersbach and his colleagues considered that the initial logarithmic mental numbers line was just a bilinear model, which was combined with two number lines of the 0–10 and 10–100 number line, with "10" breaking points.

Their studies postulated an important role of base-10 understanding on the number line estimation tasks. Based on the bi-linear number line, base-10 understanding was the dividing line of children's representing 10−100 numbers accurately. As Ebersbach et al. (2008) indicated, if the students understood 10s and accurately applied them on the 0−100 number line estimation, they would be more likely to show a linear pattern. However, for those students with the poor base-10 knowledge, their representation of 10−100 numbers lines would resemble the 0−100 logarithmic pattern.

Barth, Slusser, Cohen, and Paladino (2011) indicated the importance of using landmarks on the number line to help a student estimate the number magnitudes. They indicated that some reference numbers, usually the ratio scales on the number line, could help young children recite the number magnitudes. For instance, quarters, half, three quarters were important information for estimating the numbers on the 0–100 number line. Adults and some older children were prone to make estimation on a 0–1,000 number line in terms of landmark numbers near to 0, 250, 500, 750, and 1,000 than for other numbers. They found that multi-linear functions of the mental number line highlighted the equal scales of the multi-linear representation on the 0−10, 0−100, and 0−1,000 number lines (Barth, Slusser, Cohen, & Paladino, 2011). More accurate estimates could be obtained as long as the estimated numbers were closer to the reference midpoints (i.e., 5, 50, or 500) and quartiles (i.e., 25, 50, 75; 250, 500, 750).

Slusser, Santiago and Barth (2013) conducted an experiment to examine the development of numerical estimation of 5 to10 years old children. They found that the proportional account explained estimation patterns better than the logarithmic-to-linear-shift account for those age groups, and proposed a multi-linear function of the 0−100 number line. They claimed that as children continued to acquire crucial landmark numbers of the 0−100 number line (i.e., 25, 50, or 75), their number lines took on a linear form that was divided into two (i.e., midpoint 50) or four portions (i.e., quartiles). In their experiments, children were assessed with a number-line estimation task similar to that of Siegler's (2004, 2006). Of the 0−10, 0−100, and 0−1,000 number lines, they calculated the estimated amount to the distances to the midpoints (i.e., 5, 50, or 500) and quartiles (i.e., 25, 50, 75; 250, 500, 750). The results revealed that children were prone to use landmarks to accurately judge the position of specific numbers on the number line. As they assumed, Slusser and his colleagues found that 7–8 year-old children had developed ways to refer to marked midpoints. The mental representation referring to midpoints had a positive effect on their estimation ability. Those children using the landmarks to estimate the number magnitudes showed a lower percentage of absolute errors (PAE) than their peers who did not use the midpoints for numerical representation.

The studies of Slusser and his colleagues (2013) were important to identify the effect of using the landmarks of 10s, quarters, halves, etc. on the number line. As the students' knowledge of number knowledge grew, they would use the landmark and references number to comprehend numbers and compare the number magnitudes (Slusser, Santiago, & Barth, 2013). Therefore, children's estimation ability could be facilitated by their proportional judgments on the number line. Mathematics instructions, for instance, could incorporate these ideas by teaching children number magnitudes in terms of identifying the distances between the assigned numbers and the landmark numbers. Such instruction could be expected to elevate the students' estimation performance.

2.1.2 Estimation ability and mathematics performance. The linearity of children's number-line estimates with whole numbers correlated strongly with their accuracy of their magnitude comparisons, ability to learn solutions to unfamiliar addition problems (Booth & Siegler, 2006, 2008), and overall scores on mathematics achievement tests (Siegler & Booth 2004). The relation between numerical magnitude and standardized mathematics achievement was also found in many other studies (Bailey, Siegler, & Geary, 2014). Using number-line estimation to identify their number-magnitude knowledge was considered to be a fundamental skill for primary students (Siegler & Booth, 2006, Siegler et al., 2012).

Fazio, Bailey, Thompson and Siegler (2014) indicated that as children grew and gained more and more experience with numeracy concepts and number magnitudes, they would develop more mathematics skills and possess accurate numeracy proficiency. Fazio et al. assessed 53 fifth graders who participated in numerical-magnitude representations of fractions. They investigated the whole-numbers magnitude comparison, and number-line tasks, and obtained their mathematics and reading achievement scores. The reading scores were used to rule out the possibility of a relationship between different types of numerical-magnitude understanding and mathematics achievement. Fifth graders were studied because they had some knowledge of fraction magnitudes. Students were tested with various types of number-line estimation tasks in both 0−1,000 numbers and fraction numbers between 0−10 (Opfer & Siegler, 2007). Number magnitudes were set by dots that matched the trial with symbol numbers. Children were asked to estimate where mixed sets of blue and yellow dots belonged on the line in each trial. Those students were tested 6 months later and were found to have grown in mathematics achievement. For children using Arabic numerals on the line, their number-line estimation of fractions on all four symbolic tasks was strongly related to their mathematics achievement $r(41) = .60$. Magnitude comparison of whole numbers was also strongly related to mathematics achievement, $r(41) = .32$. The results indicated that mathematics achievement was strongly related to representation and the knowledge of numerical magnitudes both before and after controlling for symbolic magnitude knowledge $r(41) = .46$. As indicated by Fazio and his colleagues stated the extent to which the children representing the two-digit numbers closely correlation to their mathematics score. Those children with high representation ability achieved higher mathematics score; and those with low representation ability for two-digit numbers, showed more difficulties in mathematics problems solving.

As a means to investigate further, Siegler et al. (2012) conducted experiments to assess fourth graders who had developed solid estimation procedures and their ability with problem solving. They drew a conclusion that knowledge of numerical magnitudes was of great importance for young children who had not yet developed adequate strategies for problem solving. They found that improved knowledge of numerical magnitudes was positively related to higher mathematics achievement of grade 9-10 students. Children who had a better knowledge of spatial cognition of numbers (e.g., produced a linear pattern of the

0−100 number line) produced more accurate answers to computational, measurement, and other numeracy problems. Children in the primary years were less likely to acquire the number concepts and counting skills in terms of memorization and reciting practice. Instead, they relied on the various representation methods and engaging in games to approach the number magnitudes concept. Therefore, it is important to apply linear representation in early education to promote later mathematics development.

In summary, children' number magnitude understanding presented a trend from inaccurate to accurate and underwent a transition from logarithmic to a linear representation. This cognitive shift was broadly found in the various developmental stages across primary education. (e.g., from 3–10 years old). Researchers have made great efforts to disclose the developmental trend for young children, in particular 5–6 year-old children, who were in the crucial periods of kindergarten and first grade (Booth & Siegler, 2006). It has been of great interest to explore various patterns of children's mental representation of the number line, and how those number representations influenced their learning outcomes of multi-digit number magnitudes. As Siegler et al. (2012) found, the first graders who were able to more accurately estimate two-digit numbers on a 0–100 number line outperformed their peers on mathematics scores (Siegler et al., 2012). Based on those findings, I postulated that applying the landmarks on the 0–100 number, as well as helping the children to recognizing the proportions, such as 10s and quarters, would be an effective ways of improving the linear representations of numerical magnitudes (Booth & Siegler, 2008; Slusser, Santiago & Barth, 2013). Supporting the logarithmic-to-linear representation and relevant instructional intervention in a wide range of educational practice, exhibits a profound influence in facilitating children with number estimation learning, in particular those children who have learning difficulties with numerical magnitudes (Ramani & Siegler, 2008, 2014). The next session reviews the research on the methods used to teach number magnitudes to children.

2.2 Teaching Children to Understand Number Magnitudes

The linear board games, such as "Chutes and Ladders", "Candy lands", provided multiple cues to both the order of numbers and the numbers' magnitudes. Such board games provided a physical realization of the mental number line, hypothesized to be central to understanding numerical operations in general and numerical magnitudes (Ramani & Siegler, 2011). In particular, linear number board games provided children with practice at counting and at numeral identification, for example, when players were required to name the squares through which they moved (e.g., saying "6, 7" after starting on the 5 and spinning a 2). Thus, playing such games would be expected to improve counting and numeral identification skills as well as performance on tasks that required an understanding of numerical magnitudes.

Their study measured two tasks of numerical magnitude comparison and number line estimation. The results indicated that those preschoolers who were trained with linear board games, or involved in using the linear board to represent numbers, were found with significant improvements on numerical estimation and number representation. However, there was no change in estimation accuracy and linear representation pattern from pretest to posttest for the students in either the control or circular group. Therefore, Siegler and Ramani (2011) indicated that linear board games provided preschoolers with an abundance of auditory, visual, and kinetic cues that assisted them in forming accurate mental representations of the 0−10 number line. As Siegler and Ramani said (2011), playing the linear board game for roughly one hour per day increased low-income preschoolers' estimation proficiency. Playing linear number board games not only increased preschoolers' numerical knowledge, but also helped them learn from future numerical experiences. In particular, among the students of different socioeconomic status (SES), those from the low-income families gained the highest improvement from pretest to posttest (Ramani & Siegler, 2011).

Laski and Siegler (2014) extended this line of work with linear board games to assist children in developing more accurate representations of two-digit number magnitudes. In total forty-two kindergartners participated in the study. They were randomly assigned to either 10 x 10 matrix board game and control group. The children who used the 10x10 matrix board game were taught to represent two-digit numbers across a two-week period (twice per session for a total of eight sessions). The 10 x 10 matrix board games physically paralleled the 0−100 number line. Siegler and Laski indicated the case that by locating two-digit numbers on the matrix board, the kindergarteners could easily conclude that "46" was smaller than "51" because "41" was below "51" on the matrix board (Laski & Yu, 2014; Siegler, Thompson, & Opfer, 2009). Laski and Siegler (2014) found that playing on a 10 x 10 matrix board game could promote a kindergartner's estimation on the 0−100 number line. Those kindergarteners in the board game group were able to visually, verbally, and kinetically move the tokens onto the 0−100 number line so as to precisely positioned a specific two-digit numbers on the matrix board.

Using appropriate representations was essential to improve the cognitive learning of number problems for young children (Siegler & Alibali, 2005). Using an internal, mental number line can assist students to specify the problem when exposed to the numerical concept of place value, magnitudes, equivalent value, etc. Rittle-Johnson, Siegler, and Alibali (2001) conducted an experiment to instruct first and second-grade students to play number-line games to understand number magnitudes. They found that using a number line improved number knowledge of the students from pretest to posttest. Among the 74 second graders, those using the number line representation achieved a 92% correction of the arithmetic problem solving. Moreover, 53% of the students acquired the procedures of counting on and estimating the numbers on the numbers line and generated higher results for base-10 understanding (Saxe, 2004; Siegler & Alibali, 2005).

2.3 Concrete representation for place value and base-10 knowledge

Base-10 concept (Decimal base) was regarded the milestone of understanding of multi-digit number magnitudes. The Common Core State Standards highlighted these concepts as the critical instructional areas for educators to focus on with grade 1 students (Common Core State Standards Initiative, 2010, p.13). National Council of Teachers of Mathematics (NCTM) also indicated the importance to teach place-value concept to pre-K through grade 2 students (2000). Gaining insight into the underlying base 10 structure of the number system was a theory-driven trend for the importance of learning number magnitudes that may emerge in the context of estimation tasks. In order to ensure children's deep learning of place-value, educators must present these concepts in meaningful contexts with appropriate manipulatives. While young children may not be ready to understand much of the complexity of place-value, many findings suggest that teaching grouping-by-10s with manipulative has the potential to transform young children's approach to better solve the multiple-digit number magnitudes problems. Manipulatives have the potential to be powerful tools in helping children build an understanding of the base 10 number system. Children using a manipulative to represent 10s and 1s were reported with better strategies (e.g., Counting on) and accuracy in arithmetic. The students who were able to comprehend and apply the place-value correctly in the process of learning two-digit number were improved on advanced strategy use and accuracy in arithmetic over time.

There were several obstacles that must be overcome if young children intended to better understand the base 10 structure of numbers when enumerating. The biggest conceptual hurdle is shifting from perceiving enumeration as counting-by-1s to one that involves counting multi-units and leftover single units. This is especially challenging, as Fuson and Briars (1990) noted that English-speaking children, in particular, tend to conceive of multi-digit numbers as “concatenated single-digit numbers” (p. 181), rather than numbers composed of tens and ones. Fuson and Briar (1990) claimed that the most useful methods to represent number magnitudes were based on a strong place-value understanding, particularly when children were developing from working with single-digit numbers to two-digit numbers in the early elementary years. They applied the base-10 blocks to embody the English named-values system of number words, in particular, to embody the positional base-10 system of numeration. Children practiced multidigit problems of five to eight places after they could successfully add or subtract smaller problems without using the blocks. Some base-10 questions are used to assess the students’ place value concepts. The students had to identify an equation: 67 + 1285 = 67 + 12 + 8. They had to identify if the left side and the right side same or different. The comparison question was to circle the larger numbers and insert < and > symbols in the number pairs such as 681 vs. 802. The last question was the "Trade for 0” test. In the two operations of $\begin{array}{r} 1 \\ 28 \\ +36 \\ \hline 64 \end{array}$ and $\begin{array}{r} 107 \\ 2\cancel{0}\cancel{8} \\ -\ \ 62 \\ \hline 146 \end{array}$, students had to identify whether the traded “1s" were “1”, “10”, or “100”. The trade was subtracted from rather than added to the top number. However, in subtraction problems, these errors included the following: the left column was not reduced by 1 even though a trade was recorded in the right column; a trade was made even though the top number was already larger; more than one trade was made from a given column; 1 was subtracted from the traded to column; the right column received 11 rather than 10, etc. (Fuson & Briar, 1990).

To inform the students the base-10 concept, the teacher unitized various representations to introduce the arithmetic procedures of grouping by 10. One of the common ways was to use the sticks and bundles that consist of 10 sticks to represent two-digit numbers. For instance, 24 was represented with two 10 bundles and four sticks. This representation method was commonly used in the primary mathematics classroom and successfully engage the K-1 students in the funny activities of number learning and exploration. Hiebert and Wearne (1992) revealed that compared to other students who did not use bundles, students benefited from exploring the group-of-ten idea, and in doing so, developed a better place-value understanding. Students were asked to using 10 beans and carry out a variety of quantification tasks. The students were told there were ten in each bundle; one of the bundles was counted together to confirm this. Then the students were asked how many sticks there were, which was the primary interest. Researchers trained 153 first graders from six classrooms in one school with conceptualizing the base 10 instruction on place value. Pictures of bean sticks were shown and students recorded the numeral in tens/ones tables: first in tens/ones tables and then as ordinary two-digit numerals. The number 57 was interpreted as 5 groups of 10 objects with 7 left over. They solved the problem by thinking about the fractions in terms of those manipulatives: 14 plus 25 was 1 base-10 block plus 2 base-10 blocks, and then 4 unit-blocks plus 5 unit-blocks. There were a total of 16 items on the written test of place-value lessons and addition and subtraction lessons. The students in the experimental group used the bundles for demonstrating acting on and communicating the quantities. They used new textbook that provided details of how to use bundles to representing 10s. The new textbook guided the students practice the arithmetic procedures of grouping-by-10 with bundles. On the other hand, the students in the control group were just under the regular mathematics classes and used the conventional textbook. They did not receive any specific instructions on how to use bundles to represent 10s. Neither did they receive any representational ways of grouping-by-10 procedures as shown in the textbook. The students who learned how to use the bundles to represent base-10 concept performed significantly better on place value understanding than the control group. They also achieved higher on two-digit addition and subtraction with regrouping, and used strategies more often that exploited the 10s and 1s structure of the number system. Both groups of students entered instruction with some facility in counting-by-10 and understood simple story situations. However, the bundle group presented a higher ability of groupings of 10 with multidigit and they were more likely to apply this skill to solve two-digit addition problems.

Bundles and sticks were common mathematics manipulatives widely used in the mathematics problem-solving and helped the students acquire the new concepts and skills of base-10 concept. The first and second graders who learned to use bundles demonstrated the tendency of selecting the complex and effective arithmetic strategies for arithmetic problem solving, such as borrowing 10s and trading off (Kilpatrick, Swafford, & Findel, 2001). In their studies the students typically progressed from procedural learning to conceptually understanding and used more efficient ways with the bundles and sticks. The procedure of using base-10 came into play at a crucial step in the computation learning. Therefore, Kilpatrick and his colleagues expected to see that those students would show the adaptive strategies in computations across the studies. In total, 40 first graders participated in arithmetic strategic procedures for bundles studies. The participants using bundles were asked how to carry out an operation such as two-digit subtraction. They developed procedural fluency of trading off and borrowing 10s as they used their strategies to solve the subtraction problems. The students were taught the meanings of base-10 concept with bundles. During the subtraction problem-solving, using bundle and blocks were important experience helping them acquire new concepts and skills of borrowing 10s. For instance, when they learned how to solve challenging subtraction problems, the single digit number of minuend was smaller than that of the subtrahend, they depended on the ability to carry out the procedures to borrow or trade off a bundle (e.g. a "ten") readily of computational procedures. The first grades using bundles, even though they were still at very young, tended to select the borrowing 10s strategies that were well suited to those particular problems. However, those students in the control group were free to use a variety of strategies to solve the two-digit number subtraction problems. Their problems-solving ability was lower than those of the bundle group. In addition, they performed low computational proficiency than those learned to use bundles. The results indicated that the competence and fluency of computation, such as subtraction depended on the higher strategies of problem-solving. In particular, for learning two-digit numbers, using manipulatives of bundles enable the children's developmental of base-10 concept and regrouping strategies.

As the first and second-grade students began to make groupings often, they started using the language of "tens." In the studies by Van de Walle (2014), the first-grade students learned to use language that matches the bundles for grouing-by-10 concept: "Bundles of tens and singles." Then the students were guided to a general concept phase, such as "groups of tens and ones." Eventually, the students graduated to the procedural competence of using "tens" for number representation. The experimenter used grouped base-10 models of bundles to improve their "place-value understanding". Over fifty students of Grade 1 were selected to use the bundles and sticks and then constructed the numbers of ones, tens, or hundreds. For the two-digit number construction, the students used a simple two-part representation of 10s and 1s that could link to the two-digit numbers they were presented. Then the students were asked to count out bundles on the left side and sticks on the right side. They were taught in a way to understand that "one bundle and five is fifteen" when the experimenter counted out one bundle and five sticks and then "five bundles are fifteen" which means "five tens". The students were asked to repeat the procedures of number construction for other two-digit numbers in a random order but always keep bundles (e.g., 10s) on the left side. The students used bundles showed a strong ability to understand tens and even connect to the place-value concept. They were able to verbalize and construct 10s and 1s precisely for a specific two-digit numbers presented. The results also indicated that the students' place-value understanding greatly improved after playing the game of bundles and sticks (Van de Walle, 1994).

An important reason why children, especially in non-Asian countries, might struggle with base 10 understanding is because of difficulties posed by language. Research showed that children in English-speaking countries struggled to understand place value even when they could enumerate and count to high numbers, especially compared to children in Asian countries (Fuson, 1990; Ginsburg, 1989; Miura, Okamoto, Kim, Steere and Fayol, 1993). Irregularities in the English counting words, in particular the teen numbers (11-19), were believed to contribute to this lack of understanding, for the English number naming system fails to clearly distinguish between tens and ones, a feature typical of many Asian languages (Fuson, 1990; Miura et al., 1993). Miura, et al.'s (1993) studies showed that understanding 10s improve the number representation for the first graders. Asian superiority in mathematics achievement has been observed as early as the middle of the first-grade year. Asian children demonstrated outstanding performance on measures of mathematical skills such as verbal counting and base-10 understanding (Miura & Okamoto, 1989), and place-value understanding (Miura et al., 1993), even before teaching and other school-related factors came into play. This indicated that the numerical language characteristics might also be a factor in the superior mathematics performance exhibited by Asian-language speakers (Miura, Kim, Chang, & Okamoto, 1988; Miura & Okamoto, 1989; Miura et al., 1993). For instance, the English number-naming system was irregular in the teens, such as eleven, twelve and thirteen and was inconsistent with the base-10 number system. However, Asian language-speaking children (e.g., Japanese, Chinese, and Korean) learned the names of numbers between 11 and 20 were formed by compounding the "tens word" with a unit word. The numbers 11, 12, and 13 were spoken as "ten-one, "ten-two," and "ten-three." The number 20 was spoken as "two-ten(s)." Miura and Okamoto (1989) used the base-10 blocks and asked the first graders in Japan and the United States to represent two-digit numbers (e.g., canonical base-10, where number 28 was represented by the two base-10 blocks and eight unit blocks; noncanonical base-10, where number 28 was represented with only two base-10 block and eight unit blocks; one-by-one: number 28 was represented by twenty-eight unit blocks). Students were presented with five problems probing place-value understanding. They found that the Japanese students were more likely to use the canonical base 10 blocks to represent the two digit numbers. However, most of the US first grades chose unit blocks only to complete the representation task.

Miura et al. (1993) examined first-graders from France, Sweden, and Korea in their study. From each country, over twenty students participated. Significant differences in performance on the place-value understanding were found between Japanese and the U.S., French, and Swedish first graders. However, slight differences were found among the U.S., French, and Swedish first graders. Only four children (17%) in the United States, one (4%) in France, 12% in Swedish were able to answer all five questions correctly. By contrast, 10 children (42%) in Japan and 13 (54%) in Korea completed all five tasks correctly. The results indicated significant number representation difference between Japanese and American first graders. Miura et al. indicated that the incorrect constructions of two-digits numbers made by the U.S. children were found more than those students from Asian countries. This might be attributed to the lack of base-10 characteristics of the non-Asian languages.

Carpenter, Franke, Jacobs, Fennema and Empson (1998) conducted the three-year longitudinal studies among the first graders about their base-10 concepts and development of addition, subtraction algorithm. They found that the students using bundles could develop better base-10 concepts. In addition, the students used bundles for grouping and regrouping 10s practice, performed higher on the addition and subtraction algorithm rather than their peers. In total 82 students participated in the studies. They were interviewed and asked to solve the base-10 questions and arithmetic problems. The students thereafter were assigned in two different groups of experimental and control groups. The students in the experimental groups went over a series of algorithm procedures, including instruction on grouping and regrouping by 10 behaviors with the bundles and sticks. In the training session, the students were asked how many sticks there were altogether in the three bundles and the 5 loose sticks. They were also asked to solve the arithmetic word problems with the bundles and sticks. The purpose of the instruction was to improve their understanding of base-ten knowledge with each algorithm problem. For instance, when the arithmetic word problems indicated the operation of 23 +15 =? The students learned to use bundles and sticks to represent the algorithm problems. The algorithm procedure required applying grouping by 10 procedures thus the students had to show two bundles and one bundles to represent the sum of three tens and then used the single-digit number representation. The instruction and training of procedural learning of grouping by 10 were effective for the students. The results indicated that the first graders in the experimental group were found with improved base-ten concept, which was attributed to their improved procedural ability grouping and regrouping 10s ability. Another important result was that the students' base-10 concepts were closely related to their ability to solve multidigit-number addition and subtraction problems. The students with higher base-10 concept were also reported with increased multiple-digit addition and subtraction ability rather than their peers. The results supported that the development of multi-digit numbers arithmetic went together with the mastery of procedures of base-10 concept.

Jabagchourian (2008) demonstrated that canonical base-10 representation was a feasible approach for explaining place-value concepts by separating 10s and representing 1s with the exact amount of base-10 sticks and unit blocks within two-digit numbers. In his experiments, he explicitly compared the various representations of base-10 concepts. His study examined the incremental progression of numerical strategies including two-digit number representations and two-digit arithmetic among 6-year-old children. In total 85 first-graders were randomly assigned into one of four conditions: iterative, re-representation, block arithmetic, and control. Children in all conditions took a pretest that tested their two-digit representation and two-digit arithmetic abilities. Similar assessments were given five weeks later. The iterative group received a two-digit-number representation task and an arithmetic task in alternating fashion. The representation group was given an opportunity to provide three different representations of canonical base-10, noncanonical base-10, and one-by-one representation. The block-arithmetic group solved items on the arithmetic task using base-10 blocks. The results indicated that the strategies children used to represent two-digit numbers were closely connected to their abilities for arithmetic problem solving. Only those children who used canonical base-10 representations demonstrated higher strategies to solve arithmetic problems as they progressed through the sessions. However, those children using one-by-one representation tended to use just the count-all strategy for arithmetic problems. Children in the experimental conditions (e.g., canonical base-10 group) showed that their arithmetic strategies were highly contingent on problem features such as number magnitudes. Jabagchourian (2008) concluded that children faced the biggest hurdle to master place value when they had difficulties understanding how to group and decompose two-digit numbers.

Sarama and Clements (2009) examined the place-value understanding of first-grade students. The students worked with a computer base-10 manipulative. They could not move the computer blocks directly. Instead, they had to operate on symbols. Another group of students used physical base-10 blocks. The group using physical blocks was allowed to write something that represented what they did with blocks. The teachers frequently guided students to see the connection between what they did with the blocks and the numbers they wrote on paper. In comparison, the computer group understood the numbers more meaningfully, tending to connect them to the base-10 blocks. Clements found that first graders were more likely to address the mismatch problem effectively of blocks and the two-digit numbers. He concluded that students could link their representation to the two-digit numbers and understand the interrelationship between the place-value and number magnitudes.

Helmreich et al. (2011) suggested that children's estimation of two-digit magnitudes becomes more linear as they integrate tens and ones successfully. Base-10 understanding plays important roles in the beginning point of log-to-linear representation on the 0–100 number line (Helmreich, et al., 2011). From his studies, the kindergarten and first graders were found at different rate understanding two-digit numbers. In particular, this phenomenon usually occurred when the place value was instructed. This crucial mathematics concept was introduced in the K-1 classroom as the fundamental knowledge of learning number magnitude. At this point, the students began to represent the tens and ones in various ways to approach the place value concept and problems solving of multiple-digit numbers. From the findings of Helmreich, et al. (2011), the kindergarten and first grade students, who had strong place value understanding, were more likely to conceptualize the mathematics problems solving. Helmreich, et al. (2011) found that those children with clear base-10 concepts regarded 10s as another new unit, rather than a collection of 1s. Those students also performed linear representation of two-digit numbers as the second graders. The students with better place value knowledge presented accurate acquisition of number estimation of two-digit numbers, even though they were as young as 5-6 years old. The results indicated that those students with higher base-10 concept outperformed majority of the peers who were still in the phase of logarithmic number line. Helmreich, et al. (2011) concluded that mastery of base-10 concept account for the cognitive advantage of the young children in mathematics and number magnitudes learning.

2.4 Instructional Approach

The prior research on children's understanding of the base-10 concept and the number line estimation has made important contributions to this field (Helmreich, et al., 2011; Siegler & Laski, 2014; Piatt, Coret, Choi, Volden, & Bisanz, 2015). Regarding the number magnitudes task, we know that the students using base-10 representation have accurate representation, and outperformed results on number comparison task, and counting ability. The students' learning strategies, however, were not often transformed in the short periods. Thus it would be an important focus of research to examine if children's knowledge of base-10 representation would play an important role in the trajectory of the number line from immature (logarithmic) to mature (linear) representation. I postulated that the base-10 representation would be necessary and sufficient for a linear representation. In terms of promoting the internalization of base-10 spatial-numerical association (e.g., 10 powers in multiplication) a kindergarten or first graders' estimation may become more like those of the second grades.

How could instruction help children generalize and transfer their understanding of base-10 and units to number line estimation tasks and strategies? There was increasing evidence that children require explicit guidance and instruction to abstract concepts from concrete materials or to see connections between the two concepts. According to the cognitive-alignment framework (Goswami & Bryant, 2007), a theoretical framework for instructional design, even if the concrete materials were ideally designed, learning was unlikely to occur if procedures and representation did not direct children's attention to the linear representation features. Thus, educators should consider how to explicitly show children that their base-10 knowledge was beneficial in the use of number representation for mentally solving number magnitudes problems. This difference may be because many current intervention programs continue to use concrete materials throughout the early elementary school years more than regular programs. Importantly, there seemed to have the salient advantage in the shift toward number estimation for the children in the regular mathematics classroom: these children demonstrated accuracy and place-value knowledge comparable to that of children in the intervention groups. Therefore, further research is necessary to understand whether the linear representation on the 0–100 number line that the children use to solve number magnitudes estimation problems in kindergarten and first grade are predictive of estimation outcomes later in elementary school (Bailey, Siegler, & Geary, 2014). In the present study, I assumed that utilizing the combination of tens and ones within the two-digit numbers, the kindergarteners and first graders would be well educated and engaged to accurately represent the number magnitudes.

Given the range of the methods use in prior studies on children's numerical understanding. I wanted to use a methodology that would insure comparability across the conditions and different research assistants. The core intervention model (CIM) has been implemented with programs with kindergarteners and first graders to assist their mathematics (Gerber, 2003; Solari & Gerber, 2008). In particular, to facilitate lower-performing students and those from the low-income families, the core principles of CIM were used to provide different facilitations for the students of various learning levels, based on their responses and needs of the teachers' instructional interventions.

The core intervention model was a three-stair model designed to instruct the children of different levels (See Figure 1). The three-stairs was also a progressive loop model from low to high intervention based on the responses of the children. After each completion of the whole process (e.g., from stair 1 to stair 3), the teachers would lead to the beginning of the loop to repeat the original questions. The purpose of the loop was to testify the understanding and the children and make sure they have mastered the skills targeted.

Stair 1: primary interventions were provided to asks the students with some general questions. All the students began from this level to identify their understanding, like "What is this number?" The general questions aimed to draw the attention of the students and introduce them to the teaching-learning interaction and instructional pattern. If the students' responses were correct, they were considered been completed the intervention and move to some other tasks.

Stair 2: at this level the instructional intervention were applied based on the responses of the students. The teachers focused on how to correct the answers or mistakes of the students. The purpose of the intervention was to reduce cognitive load of the young children. Usually the teachers used the binary choice questions to challenge the mistake and incorrect answers of the students. For instance, "Do you think it is 45 or 55?" "Is this 45 or 55?" They would also alternatively switch the positions of the two choices to assess the understanding of the students, or using "Yes/No" binary choice question. Sometimes the teachers would repeat the questions multiples times to ensure the students were able to understand of the questions and made achievements.

Stair 3, at this level the students' learning were elevated again at another level. The goals focused on reducing the cognitive loads further. The teachers led to the correct answers directly. Based on the answers provided by the students, the teachers provided lead simple assistance to help the children's understanding. They asked the students to repeat the answers meanwhile reduced their cognitive loads furthermore. For instance, if the students still could not identify differences of number magnitudes between 45 and 55, the teachers would tell them directly: "This is number 45" and lead them to repeat the correct answers: "Read with me, forty-five". This lead simple process would continue until the students were able to read and recognized the two-digit numbers.

After the whole cycle of three stairs instruction intervention, the teachers finally led the students to the top of the loop of the model (e.g., stair 1). The teachers repeated the general question like: "What is this number"? At this point, a majority of the children would answer the question precisely and correctly. The last step was to make sure the students have acquired the number concept and were able to reply to the questions in different learning settings (Gerber, 2003).

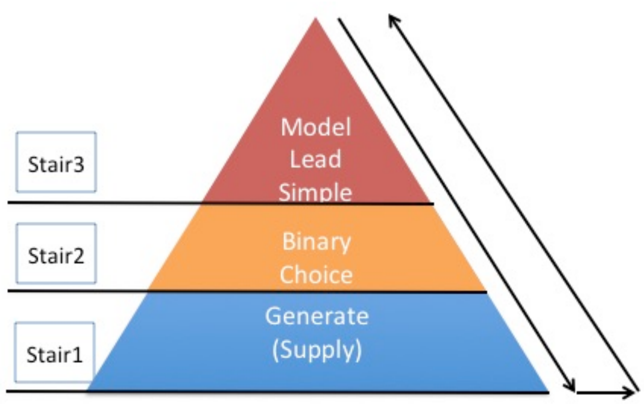

Figure 1. Core intervention model of Three-Stairs (Gerber, 2003).

Moran, Swanson, Gerber and Fung (2014) found that three-stairs instructional interventions significantly promoted the first graders' problems solving ability. The results indicated that students who received the three-stairs instructional intervention of reading comprehension practice outperformed those in the control condition in mathematics problem-solving accuracy. They conducted a study in which 72 first graders were provided with a mathematics problem solving questions. The participants were asked to address some arithmetic problems at pretest and posttest. The students under the training sessions were taught with 20 lessons. They were provided with a couple of reading comprehension task and three-stairs model to instruct them, as well correct their answers. As the standards and procedures of three-stairs model, the students who made mistakes were provided with binary choice questions. They led the students to the correct responses in terms of reducing their cognitive load. For those lower-performing students at pretest, there-stairs provided them with the fast-pace, repetitious activities that guided them to correct their answers. Finally, the low-performed students at pretest also achieved significant improvements on the arithmetic problems solving the same as those middle-to- high performed students at posttest.

2.5 Pilot Work

To address the hypothesis, in the prior study I developed the linear block game to improve number estimation ability of the grade 1 students. The students were asked to construct the two-digit number along a hypothesized 0–100 number line. Thirty-one first graders were assigned to the three groups multiple 10s, single 10 block and multiple 1s. They were asked to use the tools (e.g., base-10 and unit blocks) to construct five different two-digit numbers each session of 15 minutes across two weeks (Zhang & Okamoto, 2016). In the condition of multiple 10s, the students displayed the two-digit numbers with precise base-10 blocks and unit blocks (e.g., 28 was represented by two base-10 blocks and eight unit blocks). In that situations, there was the hypothesized landmarks of 10s along the 0–100 number line with the extension of the base-10 blocks to the expected positions. In the condition of single 10 block, the students were asked to display the two-digit numbers with just one base-10 blocks and left-over unit blocks (e.g., 28 was represented by one base-10 blocks and eighteen unit blocks). In the condition of multiple 1s, the students were asked to display the two-digit numbers with just unit blocks. No base-10 blocks were provided. Neither the base-10 concept was taught or applied. (e.g., 28 was represented by one twenty-eight unit blocks). The students also received a 0–100 number line estimation task at pretest and posttest. In the task, there was a 0–100 number line on the blank paper with 0 and 100 printed on each side as reference points.

The results were consistent with the hypothesis. The results indicated that providing persistent landmarks information of base-10 concept as did in the multiple 10s could provide the promote internal representation of number magnitudes and support the robust development of young children. The students in the multiple 10s condition were found obtained the significant improvement on number estimation task and linear representation of 0–100 number line. The students in that group successfully completed the logarithmic-to-linear representation after the training sessions, which was usually found among the second graders. However, the students in the single 10 group or multiple 1s could not achieve the comparable results. The number estimation accuracy and linear coefficient of number line were almost no changed from pretest to posttest. The students were found remain in the logarithmic representation level at the specific developmental phase of 6-8 years old. Their performances were largely found in the many prior studies that first graders were not so accurate in understanding two-digit numbers (Moeller, Pixner, Kaufmann, & Nuerk, 2009; Nuerk, Kaufmann, Zoppoth, & Willmes, 2004; Siegler & Booth, 2004, 2008). In addition, the students in multiple 10s outperformed those in other two groups across all the aspects of number estimation and linear representation, even though they were almost on the same page at pretest.

2.6 The Current Study

The present study proposed that chunking “10s” in a linear fashion effectively would lead to improvements in a child’s understanding of numerical magnitudes. The theoretical framework intended to link a base-10 number system to linear representation. Previous studies provided both quantitative and qualitative data to reveal the barrior of a logarithmic-to-linear developmental transition. In particular, for the 0−100 number line studies, those students with advanced development were more likely to acquire higher estimation ability. To eliminate the estimation performance gap and promote a fluent logarithmic-to-linear representation, there was a theoretical drive for the research to investigate the children’s development and conceptualize the learning issues.

In the present study, I continued the pilot research and applied the mental number line and base-10 block games to the two-digit learning of kindergarteners and first graders. I again adapted the mental number-line task created by Siegler and his colleagues (2004, 2006) to assess the logarithmic-to-linear cognitive shift among kindergarteners and first graders. Meanwhile, I employed base-10 questions to assess the students’ base-10 concept. I employed a bundle game as an instructional method to assess the effect on number-line estimations. My goal was to compare the two instructional approaches against each other and with a control group of students. I was interested to know how to instruct kindergarteners and first graders in two-digit numbers and to help them to complete the logarithmic-to-linear cognitive shift, as well as to develop base-10 understanding.

The students in the experimental groups would receive either the bundle or block instruction. They would be taught to represent two-digit numbers exactly with either bundles of base-10 and unit blocks. On the other hand, the control received instruction in the regular mathematics classes without receiving any specific training or interventions relevant to the numerical magnitudes. Their learning results were compared again after a period of instruction.

2.7 Research Questions

To be more specific, the research questions are:

1. To what extent do kindergartens and first graders who receive instructional interventions (i.e., the treatment group of block and bundle groups) outperform those in the control group (i.e., those without any instructional intervention) on estimation accuracy, linearity of the 0 — 100 mental number line, and base-10 score?
2. Comparing the effect of using blocks and bundles as instructional interventions, which way could help the kindergartens and first graders achieve better performance for estimation accuracy, linearity of mental number line, and base-10 score?
3. Whether or not age moderate correlation between the overall performance of estimation accuracy, linearity of number line and base-10 score prior to interventions and those afterwards?

Chapter 3

Methodology

3.1 Participants

To recruit the participants, the project was done in conjunction with a university practicum course in Southern California in the spring quarter of 2015 from April to June 2015. In the program, undergraduate students had the opportunity to work with public school elementary students. They worked with the teachers at several elementary schools in the local school districts. There were 16 undergraduate students of that course who volunteered to participate in the project and collect the data.

The student participants were recruited from ten classrooms at five elementary schools in local public school districts of Southern California. The mathematics teachers were asked to recruit kindergarteners and first graders with the lowest mathematics scores. The inclusion criteria stipulated that the recruited students should speak English fluently and not have a learning disability. From each classroom, six students were recruited by the mathematics teacher. The practicum course instructor contacted the parents and obtained permission for the study. The parents agreed to participate in the research. The consent forms were obtained with the approval of the practicum course. All the student recruits also agreed to participate in the study. In total, 60 students were recruited and began the study. However, two of the students were absent from the posttest and could not complete the study. Therefore, only 58 students participated in the entire study.

Most of the participants were bilingual speakers: English/Spanish, English/Chinese, and English/Korean. Among them 74.14 % of the participants were Latino American, 17.24 % were Caucasian American, and 6.90% were Asian American and 1.72 % African

American. The numbers of boys (n = 30, M_{age} = 5.76, SD = .16) and girls (n = 28, M_{age} = 5.59, SD = .16) were equally represented. Among them, 40 were kindergarteners (M_{age} = 5.20, SD = .41) and 18 were first graders (M_{age} = 6.73, SD =. 48)

The 58 participants (M_{age} = 5.70, SD = .84) were randomly assigned to either control group (N = 28, M_{age} = 5.67, SD = .81) or the treatment groups (N = 30) including block groups (N = 15, M_{age} = 5.73, SD = .96) and bundle groups (N = 15, M_{age} = 5.63, SD = .80) (see Table 1).

Table 1

Demographic Information of the Three Groups

	Group	Mean Age	Block	Bundle	Control
Age	*n* *M(SD)*	*58* 5.70(.84)	15 5.73(.96)	15 5.63(.80)	28 5.67(.81)
Grade	First grade	18 6.73(.48)	6 6.83(.41)	3 6.80(.72)	9 6.66(.50)
	Kindergartner	40 5.20 (.41)	9 5.00	12 5.33(.49)	19 5.21(.42)
Gender	Boys	30 5.76(.16)	9 6.00(.10)	7 5.43(.79)	14 5.77(.78)
	Girls	28 5.59(.16)	6 5.33(.82)	8 5.80(.81)	14 5.78(.86)
Ethnicity	Caucasian-American	10 5.80(.29)	3 6.33(1.15)	2 5.50(.71)	5 5.60(.89)
	Latino-American	43 5.68(.13)	10 5.70(.96)	12 5.62(.86)	21 5.71(.84)
	African-American	1 5.00	1 5.00	0 0	0 0
	Asian-American	4 5.50(.29)	1 5.00	1 6.00	2 5.50(.71)

Note: The effects of gender, grade, ethnicity, and conduction were not significantly different among the three groups at pretest.

The demographic backgrounds of the participants were representative of the population characteristics of local areas and school districts. The majority of the students were from low-to-medium income families. The State standardized test scores for these elementary schools were near the State average.

3.2 Materials and Procedures

The students were told that they would play number games. They were informed that their performance of the number line games had nothing to do with their class grades or any other academic performance. Two types of tasks were used at pretest and posttest: number-line estimation and base-10 tasks. The former task was adapted from Siegler and Booth's (2004) work and designed to assess numerical estimation ability on a 0−100 number line. The latter task was derived from Miura and Okamoto's (1988) as well as Fuson and Briar's (1990) work and designed to assess place value and base-10 knowledge. After the pretest, they in the two intervention groups participated in the training sessions. The students met with the research assistants individually for the pre- and post-tests as well as training.

3.2.1 Number-line estimation task. For the number-line estimation task, each student was provided with five pages of worksheets with 0−100 number lines on both sides of each worksheet (Siegler & Booth, 2004). The 0−100 number line was a 20-cm line drawn on the sheet with an arrow pointing from left to right. There was a "0" just below the left end and a "100" below the right end of the number line. The students were asked to estimate the position of the presented number on the number line. Each time they were presented with a two-digit number (e.g., 21), they were asked to mark the given number on the 0−100 number line and write it down below the place where they had marked (see Appendix A).

The students were asked to estimate the two-digit numbers on the 0−100 number line, one at a time. At pretest, the research assistants showed them the ten two-digit numbers (see Table 2) one at a time: 3, 9, 18, 21, 46, 65, 77, 81, 86, 96. The numbers were randomly provided. The research assistants asked: “If this is where '0' goes and this is where '100' goes, where will number (e.g., 23) go?” The students were asked to mark the number on the 0–100 number line and write the numbers below the mark. After they completed one estimation, they would proceed to the next one. They students were allowed to skip the numbers and proceed to the next practice if they so wish. This task too approximately 10 minutes to complete.

The research assistants were told not to give any feedback the answers. The students were allowed to use different learning strategies such as counting or enumerating. They were also provided with scratch papers to assist them in answering those problems. A similar process was used to assess the students at the posttest. Ten different numbers were used: 5, 7, 13, 22, 34, 53, 65, 70, 84, 92 (see Table 2).

Table 2

Order of Two-Digit Number Representations by Sessions

Pretest	3, 9, 18, 21, 46, 65, 77, 81, 86, 96				
Session 1	16,	24,	33,	41,	56
Session 2	11,	26,	31,	43,	79
Session 3	25,	46,	49,	55	92
Session 4	17,	24,	62,	84,	87
Session 5	16,	24,	33,	41,	66
Session 6	11,	26,	31,	43,	82
Posttest	5, 7, 13, 22, 34, 53, 65, 70, 84, 92				

Note: All the numbers were randomly assigned in each session

3.2.2 Base-10 task. For the base-10 task, students were presented with some number cards and blocks. They were asked five base-10 questions, one at a time. The students were asked to provide the correct responses and explanations to their answers. Each of the students worked with the research assistant for 10 minutes to complete the base-10 questions. The students would be allowed to give up answering or skip the questions that they felt were difficult.

For question No. 1, research assistants showed two number cards with number 52 and 92 written on them and asked: "How many 10s are there in number 52? …How many 10s are there in number 92?... and why?" The students were asked to provide the answers to the questions and explanations to the questions.

For question No. 2, research assistants showed a number card with the number 66 written on it and then placed out two sets of blocks. On the left side, six base-10 blocks were placed. On the right side were six unit-blocks. After that, the research assistants asked: "(Pointing to the base-10 blocks) Do these blocks belong to this '6'? (pointing to the left side of number '66')" Pointing to the unit blocks, they asked: "Do these blocks belong to this '6' (pointing to the right side of number '66')?" The students were expected to provide Yes/No answers and then explain why.

For question No. 3, research assistants showed a worksheet with an equation: $67 + 1385 = 67 + 13 + 85$. They pointed to each side of the equation and asked: "Do you think this side (i.e., the left side of the equation, $67 + 1385$) and this side (i.e., the right side of the equation, $67 + 13 + 85$) are the same or different?" The students should identify the differences and then explain why.

For questions No. 4 and No. 5, students were provided with two and three-digit number addition and subtraction problems. The research assistants showed $\begin{array}{r} {\color{red}1} \\ 58 \\ \underline{+26} \\ 84 \end{array}$ and $\begin{array}{r} 5\,{\color{red}1}0 \\ 2\cancel{6}\cancel{0} \\ \underline{-\;2\;8} \\ 2\;3\;2 \end{array}$ one at a time. They pointed to the red "1" within the operation and asked, "What does this '1' mean? Is it one, one ten, or one hundred?” The students were required to provide the correct answer and explain their response (see Table 3).

Table 3

Base-10 Understanding Questions Score Rubrics

No.	Questions	Assessment	Score (points)	Total (points)
1	How many 10s in number 52 and 92?	Five (10s); Nine (10s).	Correct response score 0.5; Correct explanation score 0.5.	1
2	Does this set belong to this 6 (left side) and this set belong to this 6 (right side)?	Yes	The same as above	1
3	Is the left side and right side the same or different? 67+1385 = 67+13+85	Different	The same as above	1
4	What does "1" mean? One, one ten or one hundred?	Ten	The same as above	1
5	What does "1" mean? One, one ten or one hundred?	Ten	The same as above	1

Note: Any incorrect, missing or skipped answers were rated as “0”.

As before, the research assistants were not allowed to provide any answers or clues to the questions. If students were unable to answer or asked for help, they would be allowed to skip the question. If the student asked for clarification, the researcher assistants merely repeated the questions to them.

After the pretest with the number-line game and base-10 questions, students were randomly assigned to the block, bundle, or control group. The students in either the block or bundle group received different instructional interventions for six sessions, twice per week. They were trained to use either blocks or bundles to construct the two-digit numbers.

Meanwhile, the control group was just exposed to the daily instructions of their school. No interventions relevant to the number-line estimations tasks were assigned. After three weeks of training, both the experimental and control groups participated in a posttest with a number-line task and a base-10 question task.

None of the participants achieved the ceiling effect on either the number-line estimation task (i.e., PAE = 0, R^2 = 100%, slope = 1) or base-10 questions (base-10 questions score = 5.0). Therefore, none of them were screened out from the present study. All the students proceeded to the next step of training sessions.

3.3 Training Sessions

The intervention groups were pulled out during regular instruction. The research assistants used the core intervention model (CIM) to work with the students during the training session. They applied the three stairs model to help the students acquire the two-digit number representation ability. The use of CIM is described in detail for each training.

3.3.1 Block group. In the training sessions for the block group, each student was provided with ten base-10 blocks sized 10 cm × 1 cm × 1 cm (i.e., length, width, and height), ten unit blocks sized 1 cm × 1 cm × 1 cm, and a solid number line. First, the students were informed that one base-10 block was equal to ten unit blocks, by counting out ten unit blocks and putting them alongside of a base-10 block (see *Figure 2.*). Thereafter, the students in the block group were asked to construct two-digit numbers with as many base-10 blocks and leftover unit blocks and lay them out along to the solid 0–100 number line. The students were asked to construct five series of two-digit numbers one at a time. The research assistants worked with students individually. The orders of the numbers provided were randomly assigned (see Table 2). The research assistants worked with students to make five two-digit numbers within 15 minutes each session. The whole training sessions lasted for 3 weeks across 6 sessions (twice per week).

At stair 1 the research assistants demonstrated that one base-10 block was equal to 10 unit-blocks. They would take out a base-10 block saying: “This is one ten,” and then they counted out ten unit-blocks. The research assistants put the two sets of blocks parallel to each other and counted: “One, two, three, four… ten.” and then asked the students: “Are they the same?” At stair 2, those students who could not answer the question were provided with a binary choice and asked: “Are there 10 blocks in each line or one hundred blocks in each line?” If a student still could not answer the question, they moved to stair 3 and the research assistants told them the correct answers directly: “There were ten blocks in each line.” This similar pattern continued until the students responded correctly. If the students still struggled, the research assistants demonstrated the answers and asked the student to repeat them.

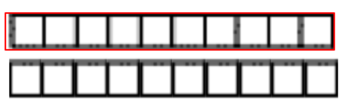

Figure 2. One base-10 block equals to ten unit-blocks

The research assistants set up an example of using base-10 blocks and unit blocks to make a two-digit number, such as 35. When the students saw the number card, they were asked to read each number aloud and then construct the numbers with the base-10 blocks provided. At stair 1, the research assistant showed the number 35 and asked students to read it aloud. If the answer was correct, “thirty-five”, students received encouragement, such as “good job”, “wonderful,” and moved to the next step. At stair 2, if they provided any answer other than thirty-five, the binary choice question was used. For instance, “Is this number 35 or 55?” The research assistants usually alternated the order of the binary choice question to check on the actual understanding of the students. If the students could not make a correct selection, they moved to stair 3 and the research assistants would say, “It is number 35; read with me, ‘thirty-five.’” The research assistant would go back to the original question again by asking: “What is this number?” to check the correct final answers.

Next, the research assistants asked: “How many 10s should be in number 35?... How many ones should be in number 35?” if the students answered correctly, they were praised and moved on to the next step. The research assistants showed ways to construct the number 35. They counted out three base-10 blocks and five unit-blocks and then put them in a line along the solid 0–100 number line. Because the length of the base-10 blocks was equal to each segment of 10s on the 0–100 number line, the students were able to notice how the base-10 blocks lined up with the number line. Meanwhile, the research assistants counted and said: “Now I’m making number 35 on the number line. I’m counting out three base-10 blocks and five unit-blocks (counting). Lay them down along the number line… One ten, two tens, three tens… One, two, three, four, five… Thirty-five.” (see *Figure 3*.) The students were allowed to interrupt or ask for clarification whenever they had questions. After the research assistants were convinced that the students understood the process of constructing two-digit numbers and did not have any further questions, they would lead them to the next step of constructing five two-digit numbers.

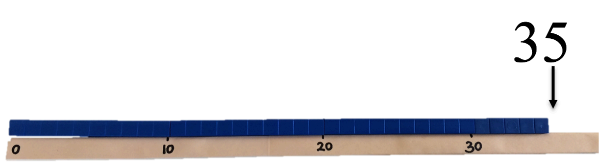

Figure 3. Block group sample representation of number 35

In each session, the research assistants showed the students different set of five two-digit numbers (see Table 2). The students were asked to construct the numbers as had been demonstrated. They repeatedly counted, moved the blocks, and constructed the numbers along the number line. The research assistants would encourage them, or correct their mistakes with the binary choice question, or simple questions. The same patterns of core intervention model were applied to instruct the students the way of constructing two-digit numbers across the 6 sessions over three weeks (see Appendix B).

3.3.2 Bundle group. The students in the bundle group were provided with sticks and bundles of sticks. Each stick was five centimeters long. The same patterns and principles of core intervention model were applied to instruct the students the ways of constructing two-digit numbers. Each session lasted approximately 15 minutes. The whole training lasted for 6 sessions across three weeks.

The procedures and protocols for the bundle group was similar to those of the block group, except no solid number line was used in bundle group, nor were the students instructed about creating linear patterns with bundles.

First, research assistants demonstrated that one bundle was equal to 10 sticks (see *Figure 4).* For instance, research assistants would take out a bundle saying: “This is one ten”. After counting out ten sticks and bundling them, they would ask:” Are they the same?” the research assistants expected the students to understand this base-10 concept and proceed to the next part.

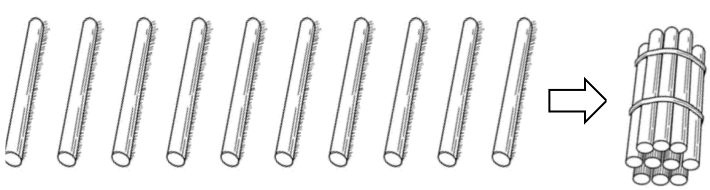

Figure 4. One bundle equals to 10 sticks

Thereafter, the research assistants demonstrated how to construct the number 35 with bundles and sticks. They showed a number card with the number 35 written on it and asked the students to read it aloud. The demonstration process was identical to that of the block group. The research assistants placed out three bundles and five sticks and put the sticks right beside the bundles in an array saying: “Now I’m making number 35. I’m counting out three bundles and five sticks (counting). Lay them down… One ten, two tens, three tens… One, two, three, four, five… Thirty-five.” (see *Figure 5.)* When the research assistants were convinced that the students understood and had acquired the skills, they proceeded to the next step.

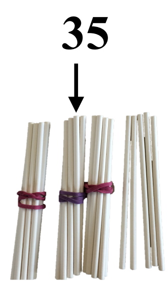

Figure 5. Bundle group sample representation of 35

Finally, research assistants showed the similar sets of five two-digit numbers as the block group. The students were allowed to ask for clarification, explanation, or help. The research assistants would encourage them, or correct their mistakes with the binary choice or simple questions. The research assistants were convinced that the students grasped the number construction ability before moving on to the next instruction.

For instance, when the students were shown with number “46”, the research assistants asked: “What is this number?” At stair 1, if the students answered correctly, they were praised and moved on to the next step. The research assistants asked: “How many 10s in number 46? ...How many ones should be in number 46?” At stair 2, if the students answered incorrectly like “five (10s)”, they would be asked again with the binary choice question: “Are they four 10s or five 10s in number 46?” After confirming that the students answered correctly, the research assistants led the students to proceed to the next step of two-digit number construction. They said: “Now you are going to make number 46 the way I did. Count out the 10s and 1s and then lay them down along the number line.” The students started counting and constructing the numbers the ways as demonstrated. At stair 3, if the students counted out five 10s to construct number 46, the research assistant would correct them like: “How many 10s should be in number 46…. Could you count out four 10s?”

3.3.3 Control group. The students in the control group received regular school instruction. Thus, they only participated in their regular mathematics classes for kindergarten or first grade. The students in the control group only participated in the pretest and posttest. They received no instructional intervention or training. In addition, the students in the control group did not receive any instruction for constructing two-digit numbers during the study.

3.4 Coding

As described in a prior study (Booth & Siegler, 2006), the researcher used a 0−20 cm ruler to measure the results of the 0−100 number-line estimations. It was a metric ruler with 0 on the left side and 100 on the right side, segmented to 100. It ranged from 0−100, as precise as 0.1. The researcher used the ruler to measure the estimated numbers on the number line. For instance, if the number to estimate is 46 and the student marked at 51.2 on the number line, the estimated number magnitude was 51.2.

3.4.1 Percentage of absolute error (PAE). The percentage of absolute error (PAE) was obtained to assess the estimation accuracy of individual students (Booth & Siegler, 2006). Taking the above data as an example, if a student was asked to estimate 46 on the

0−100 number line but marked at the location of 51.2, the PAE for number 46 for this student would be $\text{PAE}=\left|\frac{(51.2-46)}{100}\right| \times 100\% = 5.2\%$. Each student's overall PAE at both pretest and posttest is the mean PAE of the ten number estimation results. For instance, the overall PAE at pretest would be $\frac{\text{PAE}_1+\text{PAE}_2+\text{PAE}_3\ldots\ldots\text{PAE}_{10}}{\text{N}}$

PAE indicated the estimation errors for the two-digit numbers, which indicate the estimation accuracy of the two-digit number magnitudes. Reflected on the linear equation model and multivariate analyses, the decreased PAE was considered to predict the corresponding amount of increased estimation accuracy. To compare the group differences of the PAE (block group, bundle group, and control group), the individual scores were summed across and divided by the number of participants in each group. The mean PAE (n = 15) would be $\frac{\text{PAE}'_1+\text{PAE}'_2+\text{PAE}'_3\ldots\ldots\text{PAE}'_{15}}{\text{N}}$ (Siegler & Booth, 2004).

3.4.2 Linear coefficient of R^2. As described in a prior study (Booth & Siegler, 2006), a linear equation model ($y = ax + b$) was used to assess the best fitting model of the students' mental number lines. The linear coefficient R^2 and slope (i.e., slope "*a*" in the linear equation model) were obtained to assess the linearity of each student's mental number line.

A "Curve fits model" of Statistical Package for the Social Science (SPSS) Version 19.0 was applied to analyze an individual's mental number line. Both the actual and estimated number magnitudes of the specific numbers were inputted in the "Curve fit model" to obtain the best-fitting models of each student. After that, both logarithmic ($y = Inx + b$) and linear equations were obtained from the curves models program of SPSS 19.0.

I compared both the linear coefficient R^2_{linear} and logarithmic coefficient R^2_{log}. When $R^2_{\text{linear}} - R^2_{\text{log}} > 0$, the best-fitting model of an individual's mental number line was identified as a linear pattern. On the other hand, if $R^2_{linear} - R^2_{log} < 0$, the best-fitting model of his/her mental number line is logarithmic. If $R^2_{linear} - R^2_{log} = 0$ and both R^2 is larger than 80%, the best-fitting could be decided as either linear or logarithmic; or else the best-fitting model was unidentified in either pattern if both R^2 was smaller than 80% (Vitale, 2014).

3.4.3 Slope. Slope "*a*" was another index of linearity on the number line ($y = ax + b$). It shows the slope rate of the regression curve. A large R^2 together with a large slope was a strong case for a linear-fitting model of an individual's mental number line (e.g., R^2 close to 1.0 and slope close to 1.0). However, a small slope might impact the linearity of an individual's mental number line even though R^2 was large. The ideal linear model of the 0−100 number line could be $y = ax + b$ with a $R^2_{linear} = 1.00$ and $a = 1.00$. Even though the individual's R^2 was large enough ($R^2 > 80$), if the slope was very small ($a < .03$), then, it was considered weak in terms of describing the best fitting model of linear mental number line.

3.4.4 Base-10 score. For each question, the correct answer scored half point and the correct explanation scored another half point for a maximum of 1 point. That is, the students obtained full points with a correct answer and explanation. The students could just get half points if they were unable to explain their thoughts and were thereby identified as merely guessing the answers.

The base-10 score of each participant ranged from 0 to 5 points. None of the students reached the ceiling at either pretest and posttest. Six participants got a "0" score at pretest. However, none of them got a "0" score at posttest. In total, 47 answer cells were blank (of 580 total answer cells) at pretest but just 34 answers cells were empty at posttest. Those data were marked as missing data in the individual's analyses and were not be included in the final results.

Chapter 4

Results

The purpose of this chapter was to address the research questions. First, I conducted a preliminary analysis of the MANOVA at pretest. I wanted to know if there were any condition, gender or ethnicity differences at pretest. After that, I conducted multiple analyses to answer the research questions. To respond to the first research question—To what extent do kindergarteners and first graders who were under instructional interventions (i.e., the blocks and bundle groups combined) outperformed those in the control group (i.e., without receiving any instructional intervention) on estimation accuracy (PAE), linearity of the mental number line (R^2 and slope) and base-10 score? —I used the pretest and posttest to assess the effect of the instructional intervention on the kindergarteners and first graders on their PAE, the linearity of the mental number line, and base-10 scores. A one-way repeated MANOVA was applied to analyze the effect of time and condition on the students' performance.

To respond to the second research question—Comparing the effect of using blocks and bundles for instructional interventions, which would be a better approach to help the kindergarteners and first graders achieve better performance for estimation accuracy, linearity of mental number line and base-10 score? —I compared the performance results of the kindergarteners and first graders on the PAE, the linearity of the mental number line, and base-10 scores between the two treatment groups (i.e., block and bundle). I intended to find out which was the better approach to help the students improve.

To answer the third research question—Whether or not age moderates the relationship between pretest and posttest score of PAE, R^2, slope and base-10 score? —I used the linear regression model to analyze the moderate effect of age on the relation between pretest and posttest score of PAE, R^2, slope, and base-10 score for the treatment group (i.e., block and bundle groups). I was interested to see if the relation between the pretest and posttest score of PAE, R^2, slope and base-10 score would change when moderated by the age effect.

4.1 Preliminary Analyses

4.1.1 Group difference at pretest. As preliminary analyses, I conduct a one-way MANOVA to assess the condition effect on the students' performance regarding PAE, R^2, slope, and base-10 score at pretest. The results indicated that there were no significant differences found among condition group at pretest, $p = .25$.

4.1.2 Gender difference at pretest. Another one-way MANOVA was applied to examine whether there was any gender effect between boys (n =30) and girls (n =28) on the students' performance regarding PAE, R^2, slope, and base-10 score. The results indicated no gender differences were found at pretest, $p = .81$. Nor was an interaction effect of condition × gender difference found, $p = .30$.

4.1.3 Ethnicity difference at pretest. Another one-way MANOVA was applied to assess the ethnicity group differences among the students at pretest. I intended to discover if had any effect on the students learning outcomes of PAE, R^2, slope, or base-10 score. The results indicated that no ethnicity differences were found at pretest, p =. 61. Therefore, I included the results of all the conditions, genders, and ethnicity groups onto the next part.

4.2 Comparison of Treatment and Control Groups

To examine the first research question, to what extent kindergarteners and first graders who were under instructional interventions performed better than the students in control group regarding estimation ability (PAE), linearity of mental number line (R^2 and slope), and base-10 score across time, I ran a 2 × 2 repeated-MANOVA analysis (two conditions [treatment and control groups] at two times [pretest, posttest]). The results indicated that there was a significant group difference on the PAE, R^2, slope, and base-10 across time, $F(4, 53) = 5.92$, $p = .044$, $\eta^2 = .72$. There was also a significant overall improvement on the four measurements over time, $F(4, 53) = 3.39$, $p = .05$, $\eta^2 = .10$ (see Table 4). In addition, the interaction effect of condition × time was also significant, $F(4, 53) = 4.41$, $p = .05$, $\eta^2 = .32$.

Table 4

Multivariate Analyses of the Difference between Treatment and Control Groups across Time

Measure	F	p	η^2
Overall	5.92	.044	.72
PAE	7.23	.050	.62
R^2	5.33	.039	.15
Slope	5.24	.028	.28
Base-10 score	3.97	.042	.07

4.2.1 Estimation accuracy (PAE). The results indicated significant group differences in PAE across time, $F(1, 56) = 7.23$, $p =.05$, $\eta^2 = .62$ (see Table 4). Mean PAE decreased from the pretest to posttest, $F(1, 56) = 5.62$, $p =.05$, $\eta^2 = .17$, without an interaction effect of time and condition, $F(1, 56) = 8.64$, $p = .01$, $\eta^2 = .24$ (see Table 4). The mean PAE of the treatment group was significantly lower than that of the control group (see Table 5) at posttest (e.g., lower PAE indicates a higher estimation accuracy). Although the mean PAE between the treatment and control groups at pretest was comparable (15.45% vs 20.59%), the mean PAE of the treatment group was significantly lower than that of the control group (11.50% vs 15.33%) at posttest. The results of an independent samples t-test indicated the mean PAE of the treatment group was higher than that of the control group with a mean difference of 3.84, t = -2.63, p = .05, d = .98. The effect size of cohen's d is the standardized difference between the two groups. Cohen's d is the difference between the means, $M_1 - M_2$, divided by the standard deviation (e.g., if the effect size is small, d < .02; if the effect size is medium, .05 < d <. 08; if the effect size is large, d > .08).

Table 5

Mean PAE, R^2, Slope, and Base-10 Scores for the Treatment and Control Groups

		Treatment (N=30)	Control (N=28)	t	p	d
PAE	Pretest	15.45 (8.5)	20.59(3.67)	-1.72	.39	.78
	Posttest	11.50(3.95)	15.33(3.85)	-2.63	.05	. 98
R^2	Pretest	84 (.34)	.88 (.32)	-.82	.12	.12
	Posttest	.95(.12)	.89(.06)	2.48	.01	.63
Slope	Pretest	.70(.37)	.56(.26)	1.71	.24	.39
	Posttest	.85(.28)	.62(.27)	2.00	.05	.83
Base-10 score	Pretest	2.80(1.30)	2.52(1.20)	.86	.68	.22
	Posttest	3.70(.92)	3.00(1.22)	2.53	.05	.64

4.2.2 Linearity coefficient R^2. Significant group differences were found on R^2 across time, F(1, 56) = 5.33, p = .039, $ŋ^2$ = .15 (see Table 4). The mean R^2 improved over time, F(1, 56) = 22.17, p = .001, $ŋ^2$ = .95, with an interaction effect of time and condition, F(1, 56) = 8.71, p = .006, $ŋ^2$ = .24 (see Table 4). The mean R^2 of the treatment group was significantly higher than that of the control group (see Table 5). The results of an independent samples t-test indicated the mean R^2 of the treatment group was higher than that of the control group with a mean difference of .04, t = 2.48, p = .01, d = .63 at posttest. Although the group R^2 of the treatment and control groups was comparable at pretest (84% vs 88%), group R^2 of the treatment group was higher than that of control group (95% vs 89%) at posttest (see *Figures 6, 7, 8, & 9*).

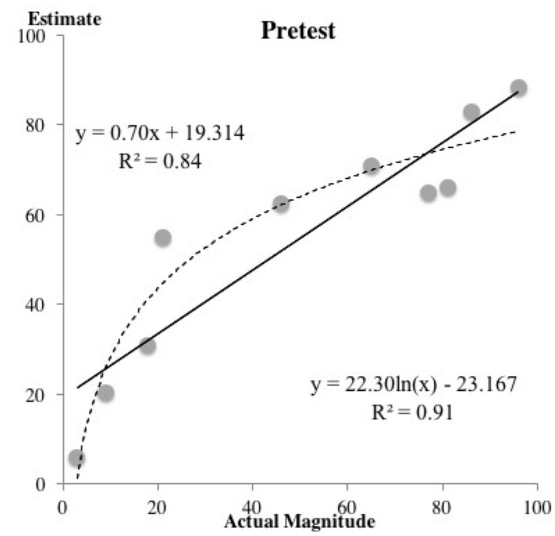

Figure 6. Best fitting model of the treatment group at pretest

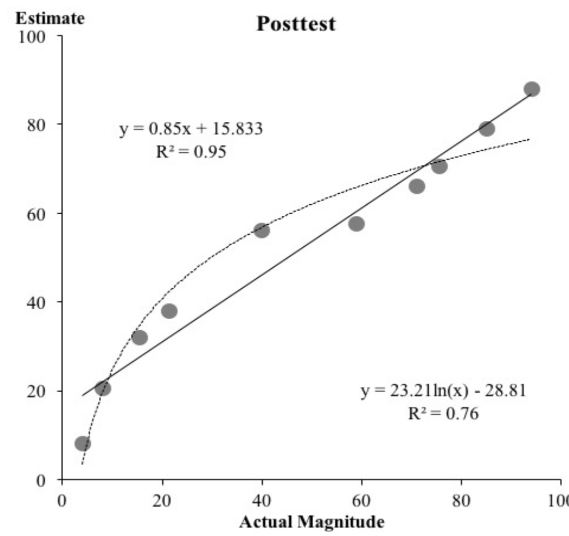

Figure 7. Best fitting model of the treatment group at posttest

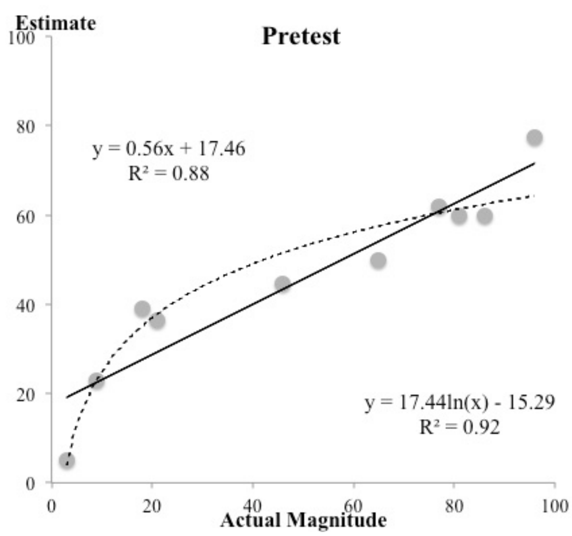

Figure 8. Best fitting model of the control group at pretest

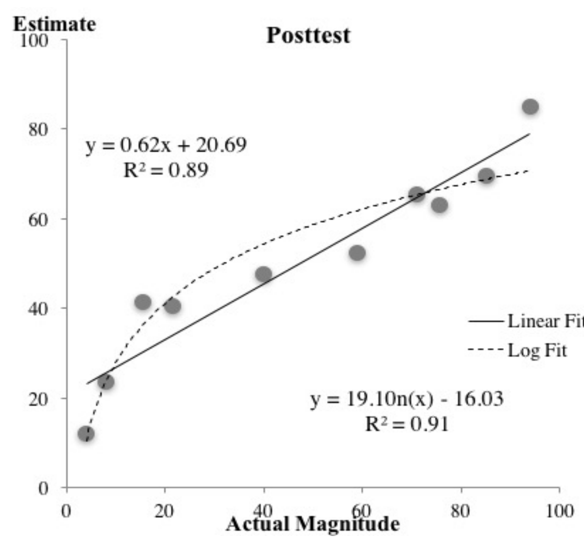

Figure 9. Best fitting model of the control group at posttest

4.2.3 Slope. Significant group differences were found on slope across time, $F(1, 56) = 5.24$, $p = .028$, $\eta^2 = .28$, (see Table 4). Mean slope improved over time, $F(1, 56) = 7.40$, $p = .004$, $\eta^2 = .12$, with a significant interaction effect of time and condition, $F(1, 56) = 4.45$, $p = .035$, $\eta^2 = .22$. The mean slope of the treatment group was significantly higher than that of the control group at posttest (see Table 5). The results of an independent samples t-test indicated the mean slope of the treatment group was higher than that of the control group at posttest with a mean difference of .23, $t = 2.00$, $p = .05$, $d = .83$. Although the group slope of the treatment and control groups was comparable at pretest (.70 vs .56), the group slope of the treatment group was higher than that of the control group (.85 vs .62) at posttest (see *Figures 6, 7, 8, & 9*).

4.2.4 Base-10 score. Significant group differences were found on base-10 score across time: $F(1, 56) = 3.79$, $p = .042$, $\eta^2 = .07$, (see Table 4). The mean base-10 score improved over time, $F(1, 56) = 12.79$, $p =.001$, $\eta^2 = .93$, with a significant interaction effect of time and condition, $F(1, 56) = 9.27$, $p = .05$, $\eta^2 = .34$. The mean base-10 score of the treatment group was significantly higher than that of the control group at posttest (see Table 5). Although the mean base-10 score between the treatment and control groups at pretest was comparable (2.80 vs 2.52), the mean base-10 score at posttest of the treatment group was significant higher than that of the control group (3.70 vs 3.00) at posttest. The results of an independent samples t-test indicated the mean base-10 score of the treatment group was higher than that of the control group at posttest with a mean difference of .70, $t = 2.53$ $p = .05$, $d = .64$.

4.3 Comparison of the Block and Bundle Group

To examine the second research question—whether students with different instructional interventions of block and bundle performed differently on estimation ability (PAE), linearity of mental number line (R^2 and slope) and base-10 score—I ran a 2 × 2 repeated-MANOVA analysis (Two conditions [block and bundle groups] at two times [pretest, posttest]). The results indicated that there was a significant group difference on the PAE, R^2, slope, and base-10 across time, $F(4, 25) = 4.54$, $p = .042$, $\eta^2 = .06$. There was also a significant improvement in the four measurements over time, $F(4, 25) = 3.29$, $p = .007$, $\eta^2 = .34$, (see Table 6). In addition, the interaction effect of condition × time was also significant, $F(4, 25) = 4.01$, $p = .008$, $\eta^2 = .49$.

Table 6

Multivariate Analyses for the Difference Between the Block and Bundle Groups across Time

Measure	*F*	*p*	η^2
Overall	4.54	.042	.06
PAE	10.24	.050	.14
R^2	4.64	.048	.02
Slope	5.62	.025	.06
Base-10 score	8.75	.008	.40

4.3.1 Estimation accuracy (PAE). The results indicated significant group differences between block and bundle group for PAE across time (see Table 6): $F(1, 28) = 10.24$, $p = .05$, $\eta^2 = .14$. The mean PAE decreased over time, $F(1, 28) = 5.65$, $p = .05$, $\eta^2 = .17$, with a significant interaction effect, $F(1, 28) = 8.63$, $p = .03$, $\eta^2 = .24$. The mean PAE of the block group was significantly lower than that of the bundle group at posttest (see Table 7) (e.g., a lower PAE indicates a higher estimation accuracy). Although the mean PAE of the block and bundle groups was comparable at pretest (17.83% vs 13.07%), the mean PAE of the block group was lower than that of the bundle group at posttest (9% vs 14%). The results of an independent samples t-test indicated the mean PAE of the block group was higher than that of the bundle group at posttest with a mean difference of 5.00, $t = -2.46$, $p = .048$, $d = .64$, (see *Figure 10*).

Table 7

Mean PAE, R^2, Slope, and Base-10 Scores for the Block and Bundle Groups

		Block (n=15)	Bundle (n=15)	t	p	d
PAE	Pretest	17.83 (8.90)	13.07(7.84)	1.56	.14	.56
	Posttest	9.00(7.55)	14.00(7.95)	-2.46	.05	.64
R^2	Pretest	.72(.31)	.93(.28)	-1.40	.17	.71
	Posttest	.97(.12)	.90(.07)	2.90	.05	.34
Slope	Pretest	.67(.45)	.74(.80)	-.47	.64	.11
	Posttest	.80(.91)	.71(.41)	2.87	.05	.13
Base-10 score	Pretest	2.97(1.50)	2.63(1.10)	.70	.49	.26
	Posttest	4.10(.96)	3.27(.71)	2.61	.01	.98

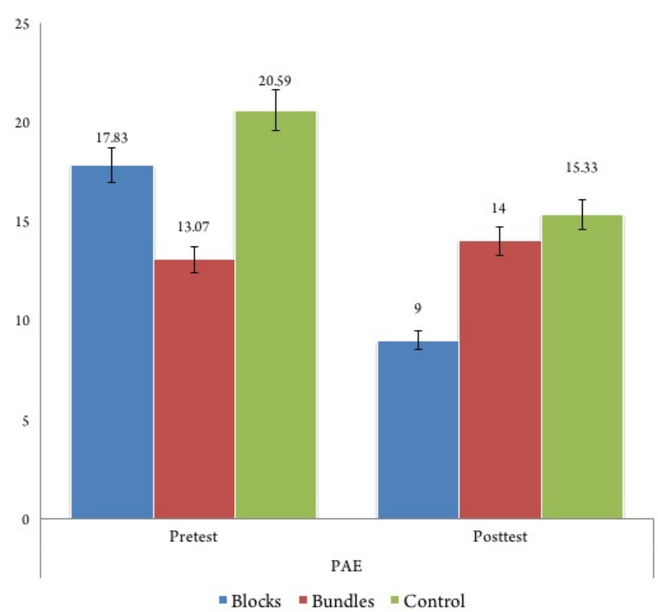

Figure 10. Percentage of absolute errors of the three groups at pretest and posttest

4.3.2 Linearity coefficient R^2. Significant group differences were found between block and bundle group on R^2 across time, $F(1, 28) = 4.64$, $p = .048$, $\eta^2 = .02$. The mean R^2 improved over time, $F(1, 28) = 7.00$, $p = .04$, $\eta^2 = .20$, with a significant interaction effect, $F(1, 28) = 8.23$, $p = .007$, $\eta^2 = .23$. (see Table 6). The mean R^2 of the block group was significantly higher than that of the bundle group at posttest (see Table 7). The results of an independent samples t-test indicated the mean R^2 of the block group was higher than that of the bundle group at posttest with a mean difference of .07, $t = 2.90$, $p = .049$, $d = .34$. Although the group R^2 of the block and bundle groups was comparable at pretest (72% vs 93%), the group R^2 of the block group was higher than that of bundle group (97% vs 90%) at posttest (See *Figures 11, 12, 13 14, &15*).

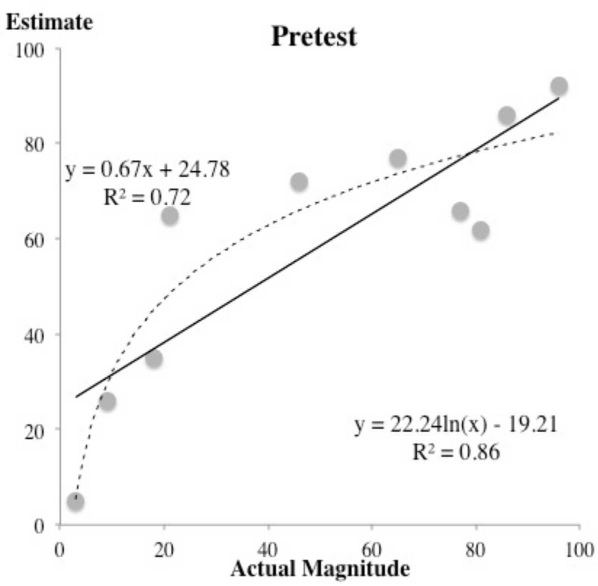

Figure 11. Best fitting model of the block group at pretest

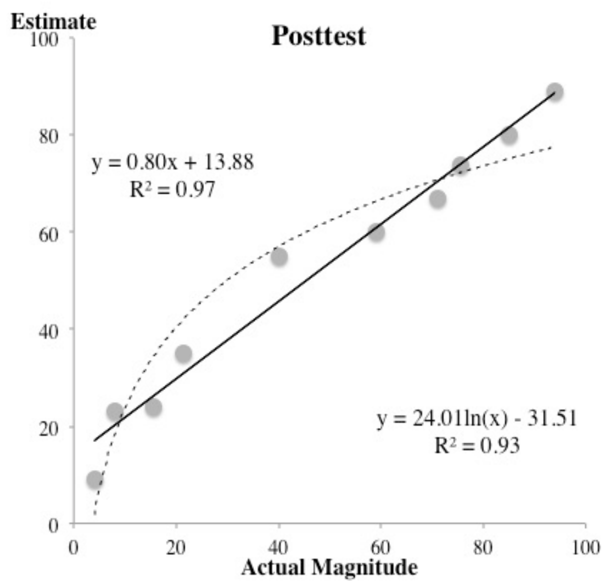

Figure 12. Best fitting model of the block group at posttest

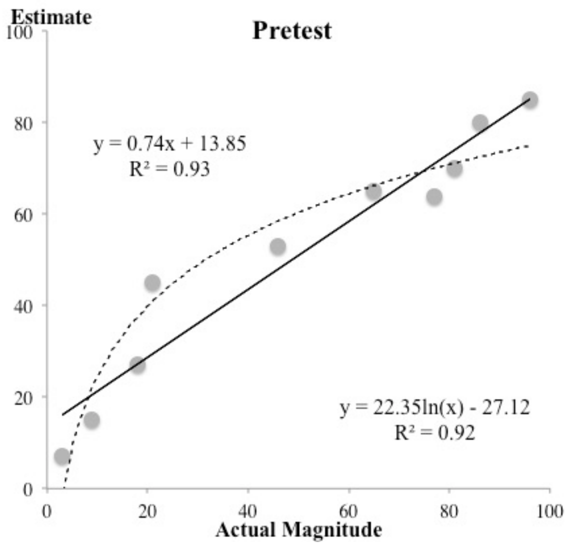

Figure 13. Best fitting model of the bundle group at pretest

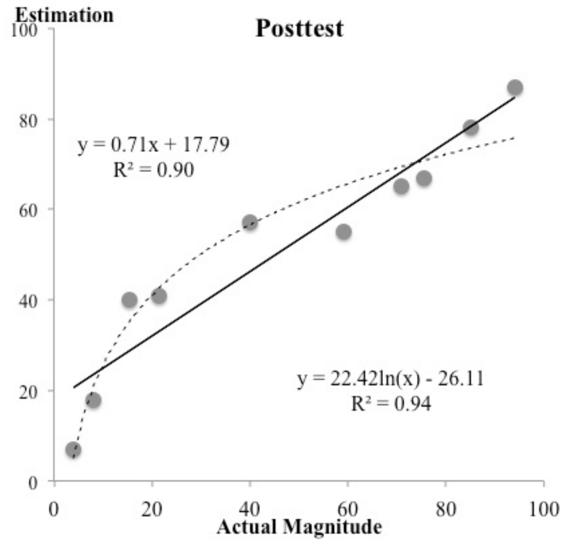

Figure 14. Best fitting model of the bundle group at posttest

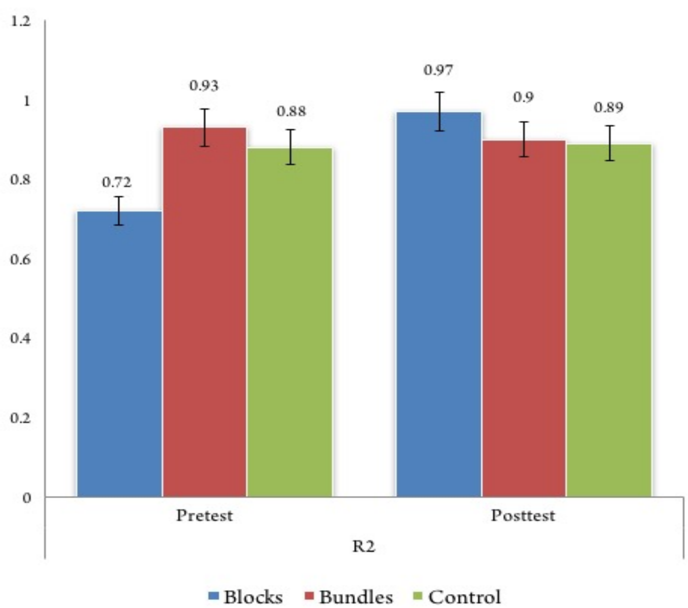

Figure 15. Linear Coefficient R^2 of the three groups at pretest and posttest

4.3.3 Slope. The results indicated significant group differences for slope between block and bundle group across time, $F(1, 28) = 5.62$, $p = .025$, $\eta^2 = .06$. The mean slope improved over time, $F(1, 28) = 7.82$, $p = .05$, $\eta^2 = .16$. (see Table 6). However, no interaction effect was found. The mean slope of the block group was significantly higher than that of the bundle group (see Table 7) at posttest. The results of an independent samples t-test indicated the mean slope of the block group was higher than that of the bundle group at posttest with a mean difference of .09, $t = 2.87$, $p = .05$, $d = .13$. Although the group slope of the block and bundle groups was comparable at pretest (.67 vs .74), the posttest group slope of the block group was higher than that of bundle group (.80 vs .71) at posttest (see *Figures 11, 12, 13 14, &16*).

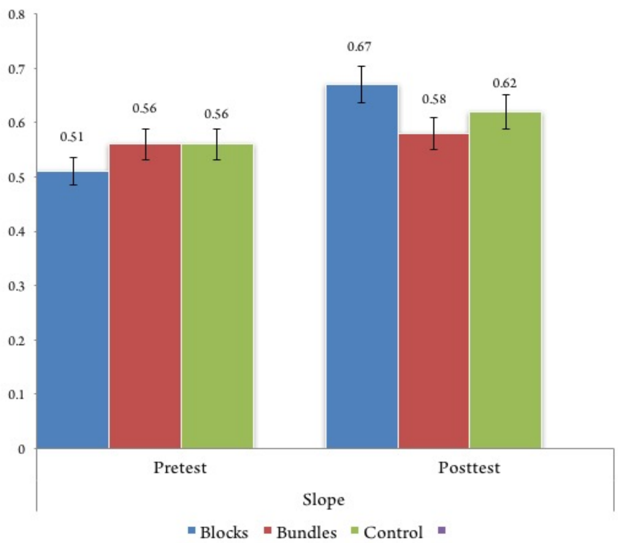

Figure 16. Slope of the three groups at pretest and posttest

4.3.4 Base-10 score. Significant group differences were found on the base-10 score between block and bundle group across time, $F(1, 56) = 8.75$, $p =.008$, $ŋ^2 = .40$. The mean base-10 score improved over time, $F(1, 56) = 14.55$, $p = .001$, $ŋ^2 = .21$, with a significant interaction effect of time and condition, $F(1, 56) = 3.27$, $p = .042$, $ŋ^2 = .02$. (see Table 6). The mean base-10 score of the block group was significantly higher than that of the bundle group at posttest (see Table 7). Although the mean base-10 score between the block and bundle group at pretest was comparable (2.97 vs 2.63), the mean base-10 score of the block group was significant higher than that of bundle group (4.10 vs 3.27) at posttest. The results of an independent samples t-test indicated the mean base-10 score of the block group was higher than that of the bundle group at posttest with a mean difference of .83, $t = 2.61$, $p = .01$, $d = .98$. (see *Figure 17.*)

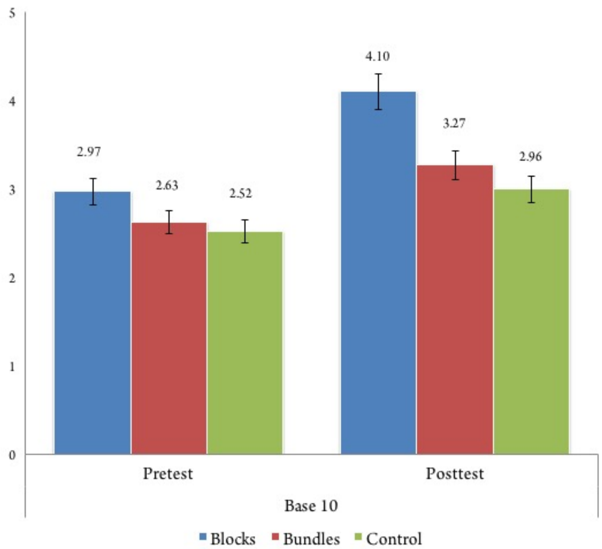

Figure 17. The base-10 scores of the three groups at pretest and posttest

4.4. The Moderator Effect of Age for the Treatment Groups

The composite score at pretest and posttest was calculated with the sum of the estimation accuracy, linearity, and base-10 score at pretest and posttest. R^2 and slope were transformed to the point score (100%). Estimation accuracy is computed as 100% - PAE. Base-10 score was transformed to 100%. A moderated linear regression analysis was run in SPSS to compare the relation of the composite pretest score and the composite posttest score for the first graders and kindergarteners.

A linear regression analysis was conducted to analyze the correlation between the composite pretest and posttest score. The distribution of the posttest score was roughly normal and the variance of the composite posttest score was not significantly different between the kindergarteners and first graders. All 30 cases of instructional intervention were included in the linear regression analysis.

The results indicated that the predictors of the composite pretest score and age were related to the outcomes of the composite posttest score. The results indicated that the composite pretest score was significantly correlated to the composite posttest score (1-tailed), Pearson $r = .37$, $p = .02$ (see Table 8). That is to say, 13.6% of the variance of the composite posttest score could be explained by the composite pretest score. The results indicated the partial correlation effect on the relationship between the composite pretest, age, and composite posttest score.

Table 8

Correlations among Composite Pretest, Posttest Score, and Age (P-Value, 1-Tailed)

		Composite pretest score	Composite posttest score	Age
Pearson Correlation *r*	Composite pretest score	-	.	
	Composite posttest score	.37 (.022)	-	
	Age	.49 (.003)	.51 (.002)	-

To assess whether the age moderated the correlation between the composite pretest and posttest score, another moderated linear regression analysis was run in SPSS to compare the relation of the composite pretest score, age, and composite posttest score for the students in the intervention group. The results indicated that the composite pretest score was also correlated to the age, $r = .48$, $p = .05$. Age was also significantly correlated with the composite posttest score, Pearson $r = .51$, $p < .001$ (See *Figure 18).*

The results of the regression analysis indicated that the overall regression equation was statistically predictive of the composite posttest score, r = .53, R^2 = .281, adjusted R^2 = .228, F(1, 27) = 5.46, p = .027. That is to say, 28.1% of the variance of the composite posttest could be explained by all the predictors (e.g., composite pretest and age) (See Table 8).

The regression equation was Ŷ (Composite posttest score) = .33 + .27 *Composite pretest score + .44*Age. The results indicated that when there was a zero composite pretest score, and age was zero, the overall performance score was .33. The results indicated that there was a main effect of age, β = .44, p = .027, and composite pretest score β = .37, p = .013. Controlling for age, the coefficient of the composite pretest score was not significantly related to the posttest score, β = .27, p = .41. Controlling the composite pretest score, for each increase in age by 1 year there was an associated increase of .44 at composite posttest score.

The results of a direct path indicated that before entering the age variable the composite pretest score was significantly correlated to the posttest score. After entering the age variable as indicated from the indirect path, the composite pretest score was non significantly correlated to the posttest score. Therefore, age moderates the relations between the composite pretest and posttest score.

(a) Direct path

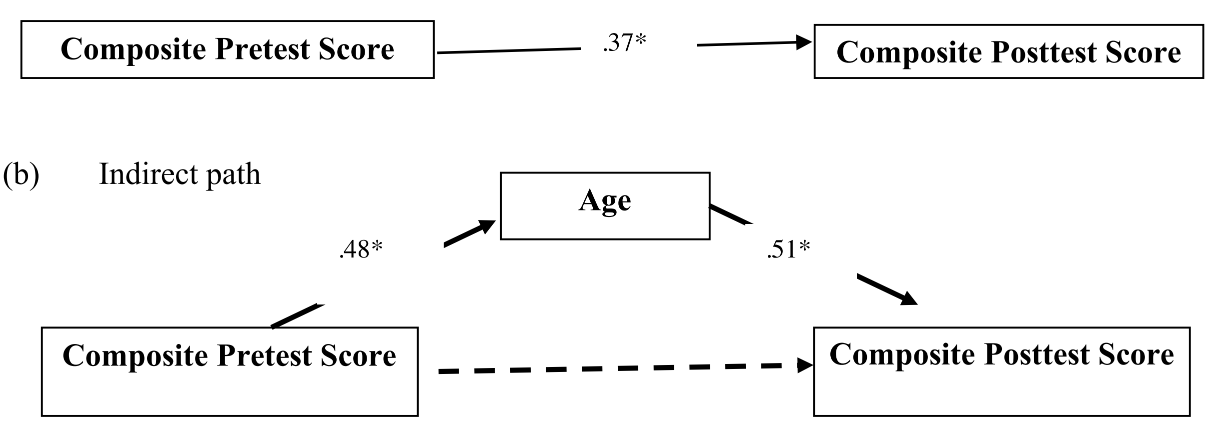

Figure 18. Linear regression model of pretest and composite

Chapter 5

Discussion

In this study, I examined the effect of instructional interventions on children's numerical learning and number magnitude understanding. First, I found that kindergartens and first graders who receive instructional interventions (i.e., the treatment group of block and bundle groups) outperformed those in the control group (i.e., those without any instructional intervention) on estimation accuracy, linearity of the 0–100 mental number line, and base-10 score. Second, I found that using the blocks manipulative over the course of 6 study sessions improved children's estimation ability and linearity of 0–100 mental number line compared to using bundles. Finally, I found that age moderated correlation between the overall performance of estimation accuracy, linearity of number line and base-10 score prior to interventions and those afterwards.

These findings extend the recent research on children's number line estimation as it highlights specific linear features of base-10 manipulatives to improve children's understanding of numerical magnitudes. This study also extends the body of literature focused on the use of manipulatives with children. Few studies elucidate the linear features of the manipulatives that contribute to children's numerical development and base-10 understanding. My findings demonstrate the value of closely examining the linear features of the 0–100 number line and base-10 knowledge of manipulatives to better understand how they impact the children's development and different types of number representations that they employed. In early elementary grades, yet findings regarding their efficacy for learning mathematics concepts were inconsistent. In the dissertation, I presented general linear principles that emerged from the cognitive developmental literature about the ways to ensure that using manipulatives with blocks could promote learning when used with kindergarten and first-grade students. I also described how base-10 knowledge instruction offers concrete representation of this linear principle in practice, which may, in turn, explain the high levels of estimation performance among the children. The linear principles of learning number magnitude with the concrete representation of base-10 knowledge presented in the dissertation should help early childhood program maximizes the benefits of number magnitudes learning in mathematics instruction.

5.1 The Effect of Instructional Interventions

In the present studies, I used the base-10 concept questions and 0–100 number line estimation as the meaningful ways to assess the development of children's numerical understanding. The students under the instructional intervention demonstrated higher levels of base-10 knowledge and numerical learning and applied those skills to solve the estimation problems, which supported the importance of early mathematics interventions. The results of the present study were an extension of the pilot study by Zhang (2014). The findings were consistent with the hypothesis that precisely using base-10 and unit blocks improved children's 0–100 number line performance compared to those in the conditions of multiple 1s or using single 10 blocks. The students on average gained a higher linearity of the 0–100 number line from pretest to posttest. Additionally, only the experimental group using multiple 10s achieved a developmental transition from logarithmic to linear within a short period of two weeks. The estimation accuracy of students in the multiple 10 condition was greater than those in the single 10 and the multiple 1 condition from pretest to posttest.

One major finding of this study was that young children using instructional manipulatives made significant learning gains in the realm of base-10 understanding and number line task compared to those who were in the regular mathematics classroom. The primary interest was to test the predictions that the kindergarten and first-grader students who received instructional interventions would outperform those in the control group. This suggested that physical manipulatives of base-10 blocks and bundles have the potential to improve children's understanding of linear representation and base-10 concepts. The kindergarten and first-grade students in the instructional intervention groups significantly improved on estimation accuracy, linear representation and base-10 knowledge after a short period of the instructions with the manipulatives. Because the children in the bundle group appeared to learn less during the course of the study, it was difficult to determine whether learning occurred to them were from the instruction. Possible explanations for these group differences were discussed in greater detail below.

This research also showed that children who used the base-10 blocks became more likely to estimate accurately and use advanced strategies (counting-by-10) after doing the activity for 6 sessions. This study was built upon Siegler and Booth's (2004) findings regarding the effectiveness of linear representation in the role of advancing children's numerical thinking. Siegler and Booth (2004) conducted "number to position" tasks to encourage K-1 students to predict and estimate the positions of two-digit numbers on the 0–100 number line. They found that children as young as 5-6 years old were able to make predictions about the number magnitudes in terms of placing the numbers on a 0–100 number line. Usually, the 5-6 year-old children were not so accurate in two-digit number estimation because they start with an initial developmental stage of logarithmic number line (Opfer & Siegler, 2007). However, children tended to use linear representation when they grew as old as the second graders.

Siegler and Laski (2014) introduced 10 × 10 matrix board game to improve the children's linear representation. Their study demonstrated that the kindergarteners used the 10 × 10 matrix board game on the course of eight sessions could improve their two-digit number estimation accuracy. While Siegler and Laski (2014) found that using 10×10 matrix board game was an effective paradigm for promoting numerical thinking, my study extended these findings (Ramani & Siegler, 2014), as I also demonstrated the usefulness of designing the blocks and bundles games within the context of core intervention instruction could accelerate the developmental pace for linear representation of young children. My results showed base-10 blocks to be a powerful method for advancing children's estimation strategy use and base-10 understanding. This dissertation goes beyond Siegler and Laski (2014) work as it outlined other possible mechanism through which the representing and counting two-digit numbers action could encourage number magnitudes learning at a high level.

Estimating the number magnitudes was regarded as one of the most crucial abilities for arithmetic and later mathematics development. In the present study, I demonstrated a novel instructional approach could advance the development of the kindergarteners and first graders who would become as accurate on estimation as second graders performed in other studies (Fazio, Bailey, Thompson, & Siegler, 2014). Applying the instructional intervention with bundles or blocks was found to promote the development of linear representation for the first graders and kindergarteners. The students learned to repeat the patterns of representing two-digit numbers in terms of combinations of 10s and 1s. This learning method could help students mentally practice and enumerate the two-digit numbers on the 0−100 number line. While they were placing and counting the blocks or bundles on the hypothesized 0−100 number line, they imitated the linear tendency mentally and behaviorally.

5.2 The Differential Effect between Block and Bundle Conditions

My second research question sought to determine whether different condition (i.e., blocks and bundles) improved children's performance on number line estimation task and base-10 questions differently. I predicted that children in both conditions would improve in their number estimation, linear representation and base-10 knowledge compared to those in the control condition. I assumed that the instructional intervention with manipulative provides the supportive and rich numerical experiences, including making children give out their responses of numbers on an interactive 0–100 number line. However, I did not anticipate significant performance differences between the block and bundle conditions since both conditions exposed children to repeated enumeration activities and required interaction with two-digit number magnitudes.

The general trend was that the children in block group did better than those in bundle group regarding the number estimation and linear representation at posttest. However, both conditions scored approximately the same in the base-10 knowledge. There were significant pairwise contrasts. Overall, children in the blocks conditions appeared to expand their number estimation more than did children in the bundle condition. It is possible that children did not have enough chances working with the blocks games to make substantial improvements. Additionally, there was explicit instruction in the block game that targeted children's linear representation. It is again possible that with such instruction both groups were able to make such cognitive improvement from logarithmic to linear representation at posttest.

However, there was nonsignificant effect of bundle condition at posttest compared to block group. The students in bundle group did not achieve the significant improvement on linear representation and numerical estimation, even though children in this group also went over a course of six sessions learning to the representation and practice of two-digit numbers with bundle manipulatives. The children in this group were not comparable on the performance of the number line estimation task as those in the block group, although they achieved the significant improvement in the base-10 knowledge.

The unexpected results of the bundle group on the number line estimation task suggested that there was less likelihood that the instructional sessions impacted children's learning of numerical magnitudes. Despite the improved base-10 knowledge, this suggested that the bundle group did not make substantial improvements on their mental number line representation. The use of the bundles manipulative required children to obtain the base-10 image but did not require children to move the objects along to the 0–100 number line by themselves (Hiebert & Wearne, 1992). It is possible that children would not have benefitted from moving bundles and sticks in the array to carry out this number representation action. Further research would explore the ideas that if unitizing bundles to resemble the ways that the physical types of 0–100 number line used in elementary classrooms would bolster mental number representation of the children. For instance, make small changes to the layout of the bundles, such as more clearly separating the base-10 segments on the 0–100 number line or labeling the side of bundles tools as "tens" and sticks as "ones".

Several possible explanations helped account for why there was a nonsignificant effect of bundle instruction. In representing the number magnitudes, children in the bundle group were less likely to connect their enumerating and counting actions to the linear representation at posttest than were children in the block group. Despite the base-10 knowledge that improved at posttest, children made no significant improvement after using the bundles in an array when representing two-digit numbers. One possibility was that the unitized bundles did not closely resemble the physical types of 0–100 number line used in elementary classrooms. It is possible that the children using bundles did not benefit from this way. It probably explained why the children were more likely to regress to logarithmic representation at posttest in the face of such visual stimuli and representing.

Another explanation for why the instruction with bundle manipulative did not positively affect children's estimation performance as compared to the blocks condition may involve the standardized design of the instructional intervention approaches. Research showed that decreasing the potentially distracting features can aid in focusing children's attention on the important mathematical concepts (Fazio, Bailey, Thompson, & Siegler, 2014; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). In the bundles group, even though the students received the similar instruction for base-10 knowledge and number line game as those in the block group, it is likely that using the bundles for instructional interventions decreased the linear features within the numbers, which might potentially distracted the attention of the children. Consequently, the block instructional intervention may have provided a simpler, less distracting model for children, which directed their attention to the crucial base-10 knowledge and linear patterns.

5.3 The Moderating Effect of Age for the Pretest and Posttest Scores

My third research question was to examine whether age moderates children's overall performance between the pretest and posttest. I predicted that children's age moderated the correlation effect between the initial performance at pretest and the that at posttest. The results on the correlation suggest that initial overall performance at pretest was less likely to impact their posttest overall performances when the students' age was considered.

The first graders were more likely to be influenced by the initial score of the pretest than the kindergarteners. Both the kindergarten and first- grade students improved approximately over 20% on the number estimation and base-10 score at posttest after the intervention sessions. These findings attested to the developmental trajectory that regardless of the grouping-by-10 manipulatives they used, older children showed learning gains much better than the young ones. Because there was a significant moderator effect on the overall performance of the number estimation and base-10 measurements at posttest, it appeared that the children with increased age may have solidified acquisition of the estimation strategy and linear representation, regardless of the manipulatives they used.

Perhaps the more important issue that arose from this results concerns the important roles that age played when interacted with the intervention effect. While this finding was expected, there were plausible explanations for this finding that children's initial abilities at pretest contributed significantly to their posttest scores. As was previously described, while there were no significant effects of condition at pretest on all the four measures, the pattern of significantly different performance scores at posttest between conditions should attributed to the effect of instructional intervention. Thus, despite the fact that overall the children under instructional intervention made significant learning gains on number line estimation and base-10 score over the course of the study, initial conditions were less likely to explain posttest differences rather than the effect of instructional intervention.

5.4 The Link between Linear Representation and Base-10 knowledge

Early numeracy skills of young children were related to mathematics achievement in future grades (Fazio, Bailey, Thompson, & Siegler, 2014). Early education generally focuses on one-to-one correspondence, counting, and number writing (Resnick & Omanson, 1987), none of which sufficiently addressed linear magnitude. Linear games were a valuable means for exploring and practicing number relationships in an effort to learn or improve mental representations (Laski & Siegler, 2014). Gameplay, which included a linear board game, and grouping-by-10 practice with blocks and bundles, provided multiple cues for forming a robust linear representation. These cues included spatial, temporal, kinesthetic, verbal, and auditory magnitude relationships if oral rehearsal and manipulation were part of the game (Laski & Siegler, 2014).

Requiring prediction of linear representation was an active teaching strategy used to increase engagement and attention. NCTM states that students "through experiences in school, should become more skilled in noticing patterns in arrangement and in using patterns to predict what comes next in an arrangement" (2000, p. 91). This also acknowledged and emphasized the importance of the teachers' behaviors and teaching quality, although the manipulatives for teaching mathematics was widely accepted, particularly in the elementary grades. Manipulatives should always be seen as a means. Simply putting concrete materials on desks or suggesting to students that they might use manipulatives would not enough to guarantee that students learn appropriate mathematics from them. Increased teachers' instruction provided a solid foundation for future mathematics success in estimation and others computational skills (Dowker, 2005; Laski & Siegler, 2014). The students should be taught to use manipulatives in a prescribed way to perform underlying mathematics concept. The teachers should also help the students see the connections among object, symbol, and the goals of mathematical learning (Kilpatrick, Swafford, & Findel, 2001).

Ramani and Siegler (2011) found that at kindergarten playing linear board games reduced the achievement gap, possibly remediating for lack of game and turn taking experience by lower achieving students. The kindergarten students in their study had at least six months' experience, often much more, of common educational experiences. My research made an extension that interventions for linear representation and base-10 knowledge produced the learning effect indicated to both kindergartners and first graders. Adaptations of this instructional method would expect to address the needs of students below the achievement performance. The outcomes were quite promising; both linear interventions and base-10 knowledge resulted in change, which was generally positive and had the greatest impact on low-achieving learners. This should also direct the attention of the teachers to the processing and understanding of linear relationship of numbers for students at different levels (Bentsong, 2013). Further research must be done to better understand the effects of the instructional intervention and refine the games and activities with a mental number line.

5.4.1 Base-10 knowledge. The children in the treatment group (i.e.., using blocks and bundles) made more accurate, unique constructions of numbers than did those in the control group. Children who used the base-10 blocks and bundles gained higher base-10 scores than did those in the control group. In addition to these findings, children in both blocks and bundles conditions also used more advanced strategies (e.g., counting-by-10) on the number line estimation task than those in the control group. There were significant pairwise differences of the base-10 score between the treatment and control group. In fact, the students in the control group performed at posttest just slightly greater base-10 understanding than they did at pretest.

It was anticipated that the act of methods of chunking 'decades' in a linear fashion would highlight the role of the 10s in multi-digit numbers and help children move faster towards seeing the set of 10 rather than merely a collection of 1s (Jabagchourian, 2008;

Miura, Okamoto, Kim, Steere, & Fayol, 1993). The findings were that both unitizing blocks and bundles manipulative improved base-10 knowledge at posttest. It appeared that the experience of using a grouping-by-10s manipulative, whatever its form and utilization, helped children conceive of multi-digit numbers as being composed of 10s and 1s, rather than just ones. In particular, when applied to the 0–100 number line, the children tended to be aware of the spatial reasoning of 10s and 1s on the number line. However, the children in the control group might not have been able to focus their attention on the grouping-by-10 mechanisms as those who used base-10 blocks or bundles.

Miura and Okamoto (1989) demonstrated that precisely representing 10s and 1s of two-digit numbers was closely related to the arithmetic ability of first graders in both Japan and the United States. The more exciting results that arose from this study concerned the improvement that children in the instructional intervention condition obtained in both the base-10 understanding and estimation ability. While this finding was expected, there were convincing findings that the children's base-10 knowledge were successful and progressed. As was previously described, while there were no significant effects of condition at pretest on the base-10 measures, the base-10 performance scores were different between intervention and control group that existed at posttest. Thus, despite the fact that overall all children made learning improvement in base-10 understanding over the course of the study, the condition by instructional intervention differences likely explained posttest differences in base-10 understanding.

5.5 Implication for Early Mathematics Intervention

Ensuring the development of mathematics competence during the primary grades was essential to later learning success. Children who had less experience or exposure to mathematical concepts and numeracy were at high risk for mathematics failure (Griffin, Case, & Siegler, 1997). Early mathematics intervention could repair deficits and prevent future deficits (Sarama & Clements, 2009). Effective mathematics and estimation instruction should include a system for monitoring student learning and adjusting instructional efforts to ensure adequate learning or accelerate it where needed. Much of the reading, writing and mathematics research on instructional intervention have occurred for distributing instructional resources to promote the greatest benefits for the greatest number of students (Bryant, et al., 2011; Solari & Gerber, 2008; Moran, Swanson, Gerber& Fung, 2014). Using core intervention model (CIM) in mathematics and estimation instruction could identify adequate screening and progress-monitoring measures. The adequacy of the CIM in mathematics can be evaluated by comparing existing instructional procedures to the elements of regular instructional programs.

In the present study, the majority of the participants were identified performed behind their peers, thus there was a need of CIM model to work with the student of that levels. The low-ability students could benefit from intervention training program based on feedback tailored to students' individual needs (Moran, Swanson, Gerber, &Fung, 2014). To standardize the teaching behaviors and interventional ways of the research assistants, the CIM was used as guidelines to direct the actions of the research assistants. Based on the procedures and rules, the research assistants taught the students two-digit number representation and corrected their mistakes and responses. The research assistants were monitored and evaluated how effective their teaching and interventions were in the following instructional plan during the training sessions, including (a) Did the research assistants clarify crucial concepts to the students and facilitate their understanding of the textbook content? (b) Did research assistants involve students in solving problems and provide monitoring for the independent work of students?

The main purpose of CIM were to classify the performance levels of the students to three stairs based on their responses to the instructional intervention. At stair 1, the research assistants provided the adequate supports and endorsement to encourage the students learning. At stair 2, effective instruction for mathematics emphasized matching the questions difficulty to the capability of the students, providing high numbers of opportunities to practice their skills and receive performance (e.g., with binary choice questions materials, different problem presentations). At stair 3, to identify the students in need, the research assistants identified the students with specific learning disabilities who performed lower and grow at a slower pace relative to their peers in learning mathematics. The core intervention model could help the research assistants provide appropriate supports and facilitations. The effective instructional interventions have added effect to understanding the students' capacity in mathematics learning.

5.6 Limitations

Despite my attempt to carefully design this study, there were certain limitations to my research that must be addressed. First, in the present study sample size was limited by the population at the educational site. The present study involved the students from ten classrooms from Southern California. Even though the demographic information of the participants was representative of the local population, the samples size is still quite limited regarding the wide population characteristics of Southern California (e.g., ethnicity, gender, age, family income, etc.). Due to the scope of this research project, I was not able to collect data from the entire recommended population sample. Additionally, in total fifty-eight participants in kindergarten and first grade, participated in the study, but the number of students in each cell was significantly smaller than the ideal. Thus, to improve reliability future studies should include samples from a variety of sites or settings to increase the population in each cell.

Second, the training time of the number representation activities were relatively short, which might not fully disclose the strength and effect of modeling the linear representation within the context of core intervention. All feedback involved either base-10 understanding or counting-by-10 strategy which provided the accuracy statements. Ramani and Siegler (2008) detected children's generation of linear representations after one hour of playing the linear board game. In this study, children played for a total of 90 minutes with 15 minutes each session and elicited results in the same pattern. Perhaps the effect of the number line game with the base-10 manipulates could be more significant given the adequate time for learning activities and interactions. While my intention was to offer children in the intervention group a beneficial experience of working one-on-one with a researcher, I should have considered extending the instruction hours, or enlarging the teachers- and-students' interaction. While the core intervention model was applied to standardize the teaching patterns for the research assistants, I should also have considered enlarging the training hours for the research assistants in the future studies, which would focus on the topics of mental number line representation and base-10 concepts, or chose microgenetic methods to instruct the number concepts to the young children (Siegler, DeLoache, Eisenberg, & Saffran, 2014).

5.7 Implication for Future Investigation

This study displayed many positive learning effects that arose from using physical manipulatives with children within the context of the core intervention model. This study had significant implications for research on children's mental number representation. One important implication of this study concerned the value of empirically testing different features of a manipulative to determine the effects on children's learning. More studies were needed that closely investigate which features of the given manipulatives promote learning. In order to better understand the mechanisms underlying the grouping-by-10 manipulatives (e.g., block and bundle), research should examine the linear features of this manipulative at an even more intensive level in future experiments. For instance, I would assess whether it was the results of linear features and interaction with the manipulatives that impacted children's learning. An important follow-up study should clarify whether the action of number learning of unitizing blocks and bundles continuously would improve young children's estimation strategy change. The follow-up sessions were essential to track students' later performance and to monitor the continuous effect of the instructional intervention. Useful longitudinal studies would examine whether the nature of the manipulatives impacted learning or advanced the estimation strategies children employ. After that, future study could focus on longitudinal studies and examine the long-term impact of such instructional methods in broader domains of mathematics.

Second, choosing the block and bundle manipulatives may not have been the only best choice for the intervention group. While my intention was to offer the children experience with mathematics that were found to enhance linear representation and base-10 knowledge, I would also select other physical or virtual manipulative (e.g., virtual math lab, number pieces, math bingo), or some reading comprehension lessons that incorporate mathematical ideas into the learning activities. Future studies could examine a range of different physical manipulatives in relation to the mathematics learning and number magnitudes understanding. One potential physical manipulative to compare with the number representation effect was the Montessori beads of mathematics instruction (Laski, Jor'dan, Daoust, & Murray, 2015). The Montessori beads was a rod of 10 beads that is unitized on one side and continuous on the other. Closely comparing this tool with the bundle or blocks manipulative could shed insights into the processes and mechanisms that promote learning.

Lastly, current findings contribute to teaching methodology for elementary mathematics instruction. Using manipulative for number magnitudes instruction was an important part of mathematics education in the classroom. This project served as a pilot for the curriculum design for elementary mathematics in the public schools. I have been designing the creative, attractive lesson plan for the preservice teachers on the annual intervention programs. My findings highlighted the positive effects of instruction intervention on children's learning, especially in the realm of number line estimation use. When the instruction s carefully designed based on the principles of cognitive psychology, it is more likely to promote the learning outcomes. Teachers should make substantial efforts to seek out appropriate, research-based learning methods and incorporate those methods into their instruction and classroom activities. Future study should integrate these new ideas into the research-based program and pre-service teachers' preparation program. Successful implementation of this type of instructional intervention would be beneficial to all students.

In summary, number estimation is a foundational skill for mathematics learning (Ramani & Siegler, 2011). Mental number line representations were important approaches for all students of mathematics. Those who started school with less prior knowledge of number magnitudes (Ramani & Siegler, 2011) typically performed poorer on academic achievement assessments. Students of low-socioeconomic (SES) status faced many academic challenges. Low academic achievement was closely related to a lack of estimation knowledge (Bailey, Siegler, & Geary, 2014; Baroody & Dowker, 2013). This was in part because low-SES students were exposed to fewer cognitive and academically stimulating activities at home and school (Ramani & Siegler, 2014). My findings, however, showed that young children from low SES backgrounds were capable of learning base-10 concepts and linear representation when taught within the context of early instructional intervention using base-10 manipulatives along the number line. These results suggested that children as young as kindergarten and first grade, including those from low SES backgrounds, should be introduced to those concepts in school. Kindergarten and first grade were critical periods for the growth of students' emergent number knowledge and estimation skills. Mental number line interventions during kindergarten and first grade would give students an additional source of support. In most cases, the lowest achieving students at the start of the intervention had the largest gains in numerical development (Ramani & Siegler, 2014). Therefore, this intervention program should focus on the skills needed. Future study should involve other education resources both inside and outside the classroom to help implement base-10 and number-line games. Providing appropriate opportunities to ensure that all learners understand and can use mathematical concepts was the goal of mathematics education and must be the focus of continued research. Future studies should also open a door for all students, parents, educators, and even educational policy makers to understand the incremental changes of numerical development of young children and the various representational methods which can be used.

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Appendix A

Transcription of Assessments

Research Assistant:

Student: Julie (pseudo name)

Number line estimation task

RA: “Now we will play a number line game. Where is 0, can you point it to the number line?”

Julie: “Here.”

RA: “Ok, where is 100? Can you point it to the number line?”

Julie: “I don’t know.”

RA: “This is where 100 is. Where does 100 go?”

Julie: “Here.”

RA: “OK.”

RA: (Showing number 21) “What is this number?”

Julie: “Twenty-one.”

RA: “This is where 0 goes, and This is where 100 goes. Where does number 21 go? Can you mark a line and write it the number 21 under the number line?”

Julie: “Yes.”

RA: “Can you put a short line on the number line showing where it is?”

Julie: “Yes.”

RA: “OK.”

RA: “What is this number (Showing number 46)?”

Julie: “I don't know.”

RA: “This is number 46.”

“This is where 0 goes, and this is where 100 goes. Where does number 46 go? Can you mark a line and write the number under the number line?”

Julie: “Yes.”

RA: “OK.”

RA: “What is this number (showing number 77)?”

Julie: “75” (wrong answers).

RA: “This is number 77. Where does number 77 goes? Can you mark a line and write the number under number line?”

Julie: “Yes.”

RA: “OK.”

RA: “What is this number (Showing 65)?”

Julie: “65.”

RA: “Can you mark it on the number line?”

Julie: “I don’t know.”

RA: “Here is where you put number 21. Where does number 65 go? Can you mark a line and write a number on the number line?”

Julie: “Yes.”

RA: “OK.”

RA: “What is this number (Showing 81)?”

Julie: “I don’t know.”

RA: “This is number 81. Here is where you put number 46. Where does number 81 go? Can you mark a line and write a number on the number line?”

Julie: “Yes.” (Mark 81 on the number line and write the number down)

RA: “OK.”

Base-10 question

Question #1:

RA: “How many 10s are in the number 52?”

Julie: “Two (10s).”

RA: “How many 10s are in the number 92?”

Julie: “Nine (10s).”

Question #2:

RA: (Showing six base-10 blocks and six unit blocks sets. Pointing to the left set of base-10 blocks) “Does this set belong to this 6 (left side)?”

Julie: “Yes.”

(Pointing to the right set of unit blocks)

“Does this set belong to this 6 (right side)?”

Julie: “Yes.”

Question #3

RA: (Showing the equation: 67+1385 = 67+13+85; T/F)

“Do you think the left part equals to this part on the right of the equation?

Julie: “I don't know.”

RA: “Is the equation: sixty-seven plus one-thousand, three-hundreds and eighty-five equals to sixty-seven plus thirteen plus eighty-five.”

Julie: “I think so.”

RA: “OK.”

Question #4

RA: (Showing the algorithm)

“What does this ‘1’ mean? Is that one, one-ten or one-hundred?”

Julie: “It’s one.”

RA: “OK.”

Question #5:

RA: (Showing the algorithm)

“What does this ‘1’ mean? Is that one, one-ten or one-hundred?”

Julie: “One-ten.”

Appendix B

Instructional Intervention

Research Assistant: RA

Student: Julie

Blocks Group

RA: “Are they the same (Putting ten unit blocks along to the base-10 block)?”

Julie: “No.”

RA: “Are there 10 blocks in each line or 100 blocks in each line?”

Julie: “10.”

RA: “Good job. Are these lines the same?”

(The research assistant demonstrates how to construct number 34 on the 0–100 number line)

RA: “Read aloud this number.”

Julie: “50.”

RA: “Is the number the number 35 or 50?”

Julie: “35.”

RA: “Good job! What is this number?”

Julie: “35.”

RA: “How many tens in 35?”

Julie: “5.”

RA: “Are there 5 tens or 3 tens?”

Julie: “3.”

RA: “Great job! How many tens in 35?”

Julie: “3.”

RA: “Wonderful!”

“Now I make number 35 on this number line; I count out three 10 blocks and 5 unit blocks (counting). Lay them down, along the number line, One ten, two tens, three tens, one, two, three, four, five.”

RA: “Read aloud this number (showing number 46).”

Julie: “46.”

RA: “Good job! What is this number?”

Julie: “46.”

RA: “How many tens in 46?”

Julie: “6.”

RA: “Are there 4 tens or 6 tens?”

Julie: “4.”

RA: “Great job! How many tens in 46?”

Julie: “4.”

RA: “Wonderful!”

RA: “Now you make number 46 the way I did.”

(Julie placed number 64 along 0–100 number line)

RA: “Are there 6 tens or 4 tens?”

Julie: “4.”

RA: “Show me 4 tens on the number line.”

Julie: (Lays out number 40).

RA: “Good job. Are there 6 ones or 4 ones?”

Julie: “6.”

RA: “Good job. Show me 6 ones on the number line.”

Julie: “Puts 6 ones beside 40 to make 46.”

RA: “Good job you made 46! “

(RA messes up blocks)

RA: “Now show me again 46 on a number line.”

(Julie lays out 46 on a number line)

RA: “Awesome!”

Bundles Group

RA: “Are they the same? (Putting 10 sticks align to a bundle)”

Julie: “No.”

RA: “Are there 10 sticks in each bundle or 100 sticks in each bundle?”

Julie: “10.”

RA: “Good job. Are these bundles the same?”

RA: “Read aloud this number (showing number 50).”

Julie: “50.”

RA: “Is the number the number 35 or 50?”

Julie: “35.”

RA: “Good job! What is this number?”

Julie: “35.”

RA: “How many tens in 35?”

Julie: “5.”

RA: “Are there 5 tens or 3 tens?”

Julie: “3.”

RA: “Great job! How many tens in 35?”

Julie: “3.”

RA: “Wonderful!”

(Research assistant demonstrate how to construct number 35 with bundles)

“Now I make number 35; I count out three 10 blocks and 5 unit blocks (counting). Lay them down, One ten, two tens, three tens, one, two, three, four, five.”

RA: “Read aloud this number (showing number 46).”

Julie: “46.”

RA: “Is the number the number 64 or 46?”

Julie: “46.”

RA: “Good job! What is this number?”

Julie: “46.”

RA: “How many tens in 46?”

Julie: “6.”

RA: “Are there 4 tens or 6 tens?”

Julie: “4.”

RA: “Great job! How many tens in 46?”

Julie: “4.”

RA: “Wonderful!”

RA: “Make number 46 the way I did.”

(Julie lays out 64 on the table)

RA: “Are there 6 tens or 4 tens?”

Julie: “4.”

RA: “Show me 4 tens on the table.”

Julie: “one ten… four tens”.

RA: “Good job. Are there 6 ones or 4 ones?”

Julie: “6.”

RA: “Good job. Show me 6 ones on the table.”

Julie: “Puts 6 ones beside 40 to make 46.”

RA: “Good job you made 46!”

(RA messes up bundles and sticks)

RA: “Now show me again 46.”

(Julie lays out 46 on a table)

RA: “Awesome!”